

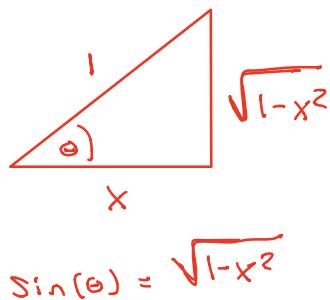
~~I~~

a) Suppose $h^{-1}(x) = \frac{x-3}{4x+5}$ is one-to-one, find $h(x)$.

$$h(x) = \frac{-5x-3}{4x-1}$$

b) If $f(x) = \cos^{-1}(x)$ what is $f'(x)$ [Show all work]

$$\begin{aligned}\cos^{-1}(x) &= \frac{1}{-\sin(\underbrace{\cos^{-1}(x)}_{\theta})} \\ &= \frac{-1}{\sqrt{1-x^2}}\end{aligned}$$



$$\sin(\theta) = \sqrt{1-x^2}$$

c) $\log_2 16 = 4$

d) $\ln \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) = \ln (1) = 0$

e) I $(1/2)^*$

II $(1/4)^*$

III 4^*

IV 2^*

II

Find Derivatives

a) $y = 5e^{\cos(x)} + \ln(\sqrt{x})$

$$y = 5e^{\cos(x)} + \frac{1}{2} \ln(x)$$

$$y' = -5 \sin(x) e^{\cos(x)} + \frac{1}{2x}$$

b) $y = 5^{x^2} + \log_{1345} x$

$$y' = 5^{x^2} \ln(5) \cdot 2x + \frac{1}{x \ln(1345)}$$

$$c) \quad y = \sqrt{x}^{\sin(x)}$$

$$\ln y = \sin(x) \ln(\sqrt{x}) = \frac{1}{2} \sin(x) \ln(x)$$

$$\frac{y'}{y} = \frac{1}{2} \left(\cos(x) \ln(x) + \frac{\sin(x)}{x} \right)$$

$$y' = \frac{\sqrt{x}^{\sin(x)}}{2} \left(\cos(x) \ln(x) + \frac{\sin(x)}{x} \right)$$

$$d) \quad y(x) = \int_e^{e^x} \ln(u) du$$

$$y(x) = F(e^x) - F(e)$$

$$y'(x) = F'(e^x) e^x = \ln(e^x) e^x = x e^x$$

III

Do integrals

a) $\int \frac{-\sin(x)}{\cos(x) \ln(\cos(x))} dx$ or $u = \ln(\cos(x))$
 $du = \frac{-\sin(x)}{\cos(x)} dx$

$\int \frac{du}{u \ln u}$ $\int \frac{1}{u} du$

$w = \ln u$ $\ln|u| + C$
 $dw = \frac{1}{u} du$ $\ln|\ln(\cos(x))| + C$

$\int \frac{1}{w} dw = \ln|w| + C$
 $= \ln|\ln u| + C$
 $= \ln|\ln(\cos(x))| + C$

$$b) \int (x+2) e^{\frac{x^2}{2} + 2x} dx$$

$$u = \frac{x^2}{2} + 2x$$

$$du = x+2 dx$$

$$\begin{aligned} \int e^u du &= e^u + C \\ &= e^{\frac{x^2}{2} + 2x} + C \end{aligned}$$

$$c) \int_0^{\log_{\pi} 5} \pi^x dx$$

$$= \frac{\pi^x}{\ln(\pi)} \Big|_0^{\log_{\pi} 5} = \frac{5}{\ln(\pi)} - \frac{1}{\ln(4)} = \frac{4}{\ln(4)}$$



a) What is the solution to the differential equation

$$\frac{dy}{dt} = ky ?$$
$$y(t) = y(0) e^{kt}$$

b) A population of bacteria starts at 1340 members. An hour later the population is observed to have grown to 1345 members. Find an expression for $P(t)$ the population at time t , where t is in hours.

$$P(0) = 1340$$

$$P(1) = 1345$$

$$P(t) \approx P(0) e^{kt}$$

$$1345 = 1340 e^k$$

$$k = \ln\left(\frac{1345}{1340}\right)$$

$$\ln\left(\frac{1345}{1340}\right) t$$

$$P(t) = 1340 e^{\ln\left(\frac{1345}{1340}\right) t}$$

c) Is the relative growth rate for part(b) positive or negative, give a reason.

Positive $\ln\left(\frac{1343}{1340}\right)$

↑

> 1