

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write **your full name** on every piece of paper that will be uploaded to gradescope. Do all problems. **Start each problem on a new page.** Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work. Only use techniques from sections 1.1-5.5.**

1. Show all work

(a) Suppose $g(x) = \frac{x-3}{4x+5}$ is one-to-one find $g^{-1}(x)$.

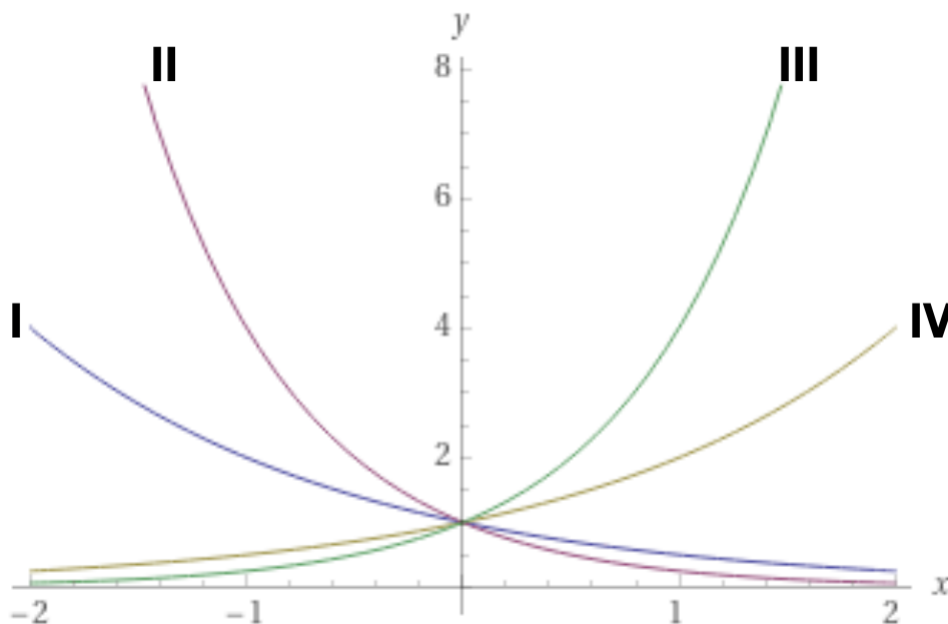
(b) If $f(x) = \cos^{-1}(x)$ find $f'(x)$.

(c) $\log_2 16 =$

(d)

$$\ln\left(\lim_{x \rightarrow 0} \frac{\sin(x)}{x}\right) =$$

(e) Match I,II,III,IV with $y = (1/2)^x$, $y = 2^x$, $y = 4^x$, $y = (1/4)^x$



2. Find derivatives of the following functions (show all work)

(a)

$$y = 5e^{\cos(x)} + \ln(\sqrt{x})$$

(b)

$$y = 5^{x^2} + \log_{1345}(x)$$

(c) Use logarithmic differentiation to find the derivative:

$$y = \sqrt{x}^{\sin(x)}$$

(d) Invoke the fundamental theorem of calculus to find the derivative:

$$y(x) = \int_e^{e^x} \ln(x) dx$$

3. Find the integrals

(a) Be patient with this one:

$$\int \frac{-\sin(x)}{\cos(x) \ln(\cos(x))} dx$$

(b)

$$\int (x+2)e^{\frac{1}{2}x^2+2x} dx$$

(c)

$$\int_0^{\log_{\pi}(5)} \pi^x dx$$

4. (a) What is the solution to the differential equation

$$\frac{dy}{dt} = ky$$

(b) A population of bacteria is observed to have 1340 members. An hour later the population is observed to have increased by 5 members. Find an expression for $P(t)$ the population at time t (in hours).

(c) Is the relative growth rate for part(b) positive or negative, give a reason.
