

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write **your full name** on every piece of paper that will be uploaded to gradescope. Do all problems. **Start each problem on a new page.** Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work. Only use techniques from sections 1.1-4.5.**

1. (a) Describe a region whose area is equal to

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \tan\left(\frac{i\pi}{4n}\right) \frac{\pi}{4n}$$

- (b) Find

$$\int_{-1340}^{1345} f(x) dx$$

given

$$\int_{1340}^{1345} 2f(x) dx = 10 \text{ and } f(-x) = -f(x).$$

- (c) Find

$$\int_0^3 x + \sqrt{9 - x^2} dx$$

Hint: Use geometry.

2. (a) Using the limit definition of the integral. Find

$$\int_0^3 5x^2 - 6 dx$$

These formulas may be useful

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

- (b) Use $n = 4$ subintervals and right endpoints to estimate.

$$\int_0^{2\pi} \sin(x)\cos(x) dx$$

How accurate is your estimate?

3. (a) If the velocity of a particle moving in a straight line is given by $v(t) = \sin(t)$ find the displacement over the time interval $(0, \pi)$.

- (b) Find the total distance traveled by the particle over the same time interval.

- (c) Bound the integral $\int_0^3 \sin^2(x) dx$ above and below using the comparison property.
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4. (a) Find

$$\frac{d}{dx} \int_{\cos(x)}^{\sqrt{x}} \frac{1}{\tan^2(t) + 5} dt$$

- (b) Find the average value of $f(x) = x^2$ over the interval $(0, 3)$. Draw a graph of $f(x)$ indicating the average value and its relation to the area under the curve of $f(x)$.
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5. Evaluate the following

(a)

$$\int (3 - x)^{10} dx$$

(b)

$$\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

(c)

$$\int_0^{\pi/8} \sec(2x)\tan(2x) dx$$