a) Newton's Method will always converge.
   - FALSE

b) Does the point \( x = 0 \) have any significant attributes for \( f(x) = x^3 + 3 \)?
   - Point of inflection

c) Suppose you know \( f(x) = 0 \) when \( x = 1 \).
   If you are using Newton's Method on \( f(x) \) and let \( x_1 = 1 \), what is \( x_2 \)?

\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}
\]

\[
x_2 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{0}{3} = 1
\]

d) Find the most general antiderivative for
   \( f(x) = \sin(x) + 2x \)

\[
f(x) = -\cos(x) + x^2 + C
\]
Sketch the curve \( f(x) = \frac{1}{1 + x^2} \)

**Domain:** all real numbers

**Asymptotes:** No vertical, since domain is IR

**Horizontal:**

\[
\lim_{x \to \pm \infty} f(x) = 0
\]

**Symmetry:**

\[ f(-x) = \frac{1}{1 + (-x)^2} = \frac{1}{1 + x^2} = f(x) \]

Even function.

**Intercepts:**

**X-intercepts**

\[ 0 = f(x) = \frac{1}{1 + x^2} \]

\[ 0 = 1 + x^2 \]

Not possible

**Y-intercepts**

\[ f(0) = \frac{1}{1 + 0} = 1 \]
**Inc/Dec**

\[ f(x) = \frac{1}{1+x^2} \cdot (1+x^2)^{-1} \]

\[ f'(x) = -\frac{2x}{(1+x^2)^2} \]

\[ f'(x) = 0 \quad \text{when} \quad x = 0 \]

\[ f' \]

\[ + \quad \bigcirc \quad - \]

\[ f \quad \text{is increasing on} \quad (-\infty, 0) \quad \text{and has a local max of 1 at} \quad x = 0 \quad \text{decreasing on} \quad (0, \infty) \]

**Concavity**

\[ f(x) = \frac{1}{1+x^2} \]

\[ f'(x) = -\frac{2x}{(1+x^2)^2} \]

\[ f''(x) = \frac{-2(1+x^2)^2 + 2x \cdot 2(1+x^2) \cdot 2x}{(1+x^2)^4} \]
\[
\frac{-2(1+x^2) + 8x^2}{(1+x^2)^3}
\]

\[
f''(x) = \frac{6x^2 - 2}{(1+x^2)^2}
\]

\[
f''(x) = 0 \quad \text{when} \quad 6x^2 - 2 = 0 \quad \Rightarrow \quad x = \pm \frac{1}{\sqrt{3}}
\]

\[
\begin{array}{cccc}
+ & - & + & \\
\hline
& -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \\
\end{array}
\]

$f$ is concave up on $(-\infty, -\frac{1}{\sqrt{3}})$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$

$f$ is concave down on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
Find two positive numbers whose sum is 300 and whose product is a maximum.

\[ x + y = 300 \quad \Rightarrow \quad y = 300 - x \]

\[ P = xy = x(300 - x) \]

\[ = 300x - x^2 \]

\[ P' = -2x + 300 \]

\[ 0 = -2x + 300 \]

\[ x = 150 \]

\[ y = 150 \]

\[ P'' = -2 \quad \Rightarrow \text{concave down} \]

\[ x = 150 \text{ is maximum} \]
a) Using Newton's method on $\sin(x) = x$

Find $x_2$ given $x_1 = \frac{\pi}{2}$

$f(x) = \sin(x) - x$

$f'(x) = \cos(x) - 1$

$x_2 = x_1 - \frac{f'(x_1) - x_1}{f''(x_1) - 1} = \frac{\pi}{2} - \frac{\sin\left(\frac{\pi}{2}\right) - \frac{\pi}{2}}{\cos\left(\frac{\pi}{2}\right) - 1}

= \frac{\pi}{2} - \frac{1 - \frac{\pi}{2}}{-1} = \frac{\pi}{2} - \left(\frac{\pi}{2} - 1\right) = 1$

b) Set up the formula for $x_{n+1}$ if you were seeking to find a max/min of $f(x)$

Want max/min of $f(x)$

Find $f'(x) = 0$

Perform Newton's Method on $f'$

$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$
5. Given the acceleration of a particle moving in a straight line is \( a(t) = 6t \).

Find the position function given \( V(0) = 1 \) and \( S(1) = 0 \).

\[ a(t) = 6t \]

\[ V(t) = 3t^2 + C \quad V(0) = 1 \]

\[ V(t) = 3t^2 + 1 \]

\[ s(t) = t^3 + t + D \quad s(1) = 0 \]

\[ s(t) = t^3 + t - 2 \]