INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your full name, (2) 1345/Exam 3, (3) lecture number/instructor name and (4) SPRING 2020 on the front of your 8.5” × 11” paper. Make a grading table for 4 problems and a total. Sign and date the honor code. Do all problems. Start each problem on a new page. Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Justify your answers, show all work.

1. (24pts) The following parts of this problem are not related.

   (a)(12pts) Suppose the function \( h^{-1}(x) = \frac{4x - 1}{2x + 3} \) is one-to-one, find \( h(x) \).

   (b)(12pts) Suppose \( f(x) = \int_{e}^{x} e^{t} \ln(t+2) \, dt \) for \( x > 0 \). Find \( (f^{-1})'(0) \). [Hint: Recall that \( \frac{d}{dx} \int_{a}^{x} g(t) \, dt = g(x) \).]

2. (28pts) Start this problem on a new page. The following parts are not related.

   (a)(12pts) In 1984 Linda Hamilton saves 200g of an unknown isotope from a failed cyborg assassin. In 1991 there remains 120g of the unknown isotope. (Vintage problem!) Answer the following questions:
   
   (i)(6pts) What is the relative decay rate of the unknown isotope? No need to simplify your answer.
   
   (ii)(6pts) How much of the isotope will Linda have in 2012 when Skynet© decides to strike? No need to simplify your answer.

   (b)(12pts) At what point on the curve \( y = 1 + 2e^{x} - 3x \) is the tangent line parallel to the line \( 3x - y = 5 \)? Specify both the \( x \) and \( y \) coordinates.

   (c)(4pts) Multiple Choice: Consider the function \( g(x) = \ln(x^2 + x + 1) \) for \( -1 \leq x \leq 1 \). The function \( g(x) \) has an absolute minimum at \( (x, y) = (-1/2, \ln(3/4)) \) and an absolute maximum at \( (x, y) = \) see choices below (write answer in bluebook). 
   
   (No justification necessary, choose only one answer – copy down the entire answer in your bluebook.)
   
   (A) \( (x, y) = (-1, 0) \)  (B) \( (x, y) = (-1, \ln(3)) \)  (C) \( (x, y) = (1, 0) \)  (D) \( (x, y) = (1, \ln(3)) \)  (E) \( (x, y) = (-1/2, \ln(3/4)) \)

PROBLEMS #3 & #4 ON THE NEXT PAGE
3. (24pts) Start this **problem** on a new page. The following **parts** are not related.

(a)(12pts) Find the antiderivative \( \int \frac{5^x}{1+5^x} \, dx \).

(b)(12pts) Use logarithmic differentiation to find the derivative of \( y = \frac{\sqrt{x}e^{x^2}}{(x^2 + 1)^{10}} \).

4. (24pts) Start this **problem** on a new page. The following **parts** are not related.

(a)(10pts) Evaluate \( \int \frac{\ln(x^2e^{\sqrt{x}})}{x} \, dx \).

(b)(10pts) Evaluate \( \int_1^2 \frac{e^{1/x^3}}{x^4} \, dx \).

(c)(4pts) **Multiple Choice:** Which of the five choices given below is equivalent to \( y' \) if \( y = (\sin x)^{\ln(x)} \)? (No justification necessary, choose only one answer – copy down the entire answer in your bluebook.)

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\begin{align*}
(A) \quad & \ln(x) \sin(x)^{\ln(x)-1} \\
(B) \quad & -\sin(x)^{\ln(x)} \cos(x)^{1/x} \\
(C) \quad & \ln(x)^{\sin(x)} \left[ \frac{\ln(x)}{x} + \ln(x) \tan(x) \right] \\
(D) \quad & \sin(x)^{\ln(x)} \left[ \frac{\ln(x)}{x} + \ln(x)^{\cot(x)} \right] \\
(E) \quad & \frac{\ln(x)^{\sin(x)}}{x} + \ln(x) \tan(x)
\end{align*}
\]