INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your full name, (2) 1345/Exam 2, (3) lecture number/instructor name and (4) SPRING 2020 on the front of your bluebook. Make a grading table for 4 problems and a total. Do all problems. Start each problem on a new page. Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Justify your answers, show all work.

1. (24pts) The following problems are not related. For this problem you may or may not find the following formulas useful:

\[
\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \text{and} \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.
\]

(a)(12pts) Find the value of the sum \(\sum_{i=1}^{n} i(2i^2 - 3)\) in terms of \(n\). (Do not take any limits.)

(b)(12pts) Evaluate the limit by any method: \(\lim_{n \to \infty} \sum_{i=1}^{n} \left(2 - \frac{4i^2}{n^2}\right) \frac{2}{n}\)

2. (28pts) The following problems are not related.

(a)(12pts) Find an approximation to the integral \(\int_{0}^{6} (x^2 - 5x + 4) \, dx\) using a Riemann sum with \(n = 6\) subintervals of equal width and using left endpoints, i.e. find the \(L_6\) approximation of the integral. (Do not take any limits.)

(b)(12pts) If \(\int_{1}^{5} f(x) \, dx = 24\) and \(\int_{5}^{7} f(\Theta) \, d\Theta = 3.6\), find \(\int_{1}^{4} f(x) \, dx\).

(c)(4pts) Which one of the choices given below is equivalent to \(\lim_{n \to \infty} \sum_{i=1}^{n} \sin \left[2 \left(\pi + \frac{\pi i}{n}\right)\right] \frac{\pi}{n}\)? Choose only one answer. No justification necessary, copy down the entire answer. If you do not copy down the entire answer, points will be deducted.

(A) \(\int_{0}^{\pi} \frac{\sin(2\pi + x)}{x} \, dx\) \quad (B) \(\int_{0}^{2\pi} \sin(2x) \, dx\) \quad (C) \(\int_{0}^{\pi} \sin(2x + \pi) \, dx\) \quad (D) \(\int_{-\pi}^{\pi} \sin(2x) \, dx\) \quad (E) \(\int_{\pi}^{3\pi} \frac{\sin(2x)}{2} \, dx\)

PROBLEMS #3 & #4 ON THE OTHER SIDE
3. (24pts) The following problems are not related.
(a)(12pts) Evaluate the definite integral: \[ \int_{0}^{3} |x^2 - 4| \, dx. \]
(b)(12pts) If 0 \(\leq x \leq 2\), find numbers \(m\) and \(M\) such that \(m \leq \int_{0}^{2} \sqrt{x^3 + 1} \, dx \leq M\). Justify your answer.

4. (24pts) The following problems are not related.
(a)(10pts) Evaluate the definite integral: \[ \int_{0}^{\sqrt{\pi}} x \cos(x^2) \, dx \]
(b)(10pts) The position function of Annie the ant is given by \(s(t) = \int_{0}^{t^2} \sin(x^4) \, dx\), where \(t \geq 0\) is in seconds, find Annie’s velocity at any time \(t\).
(c)(4pts) Suppose the average value of \(h(x)\) on the interval \([1, 4]\) is \(h_{ave} = \pi\), then which choice below is true? (\underline{No justification necessary - Choose only one answer, copy down the entire answer})

- (A) \[ \int_{5}^{20} h(x/5) \, dx = 3\pi/5 \]
- (B) \[ \int_{5}^{20} h(x/5) \, dx = 2\pi \]
- (C) \[ \int_{5}^{20} h(x/5) \, dx = 15\pi \]
- (D) \[ \int_{5}^{20} h(x/5) \, dx = 8\pi \]