1. (24pts) The following problems are not related.
   (a)(12pts) Find all the intervals on which \( f(x) = \frac{x}{x^2 + 1} \) is increasing or decreasing. Give your answer in interval notation. Show all work.
   (b)(12pts) Find and classify any critical point(s) of \( f(x) = x - 2\sin(x), \ 0 \leq x \leq \pi \). Classify the critical point using the 2nd Derivative Test and give your answer in the form \((x, y)\).

2. (28pts) The following problems are not related.
   (a)(12pts) In this problem, you will find two positive integers \( x \) and \( y \) such that the sum of the first number and four times the second number is 1000 and the product is as large as possible. Answer the following questions:
   (i)(4pts) Is this a minimization or maximization problem? Write down a function in terms of the two variables \( x \) and \( y \) that you would minimize (or maximize).
   (ii)(4pts) Use the given information to write an equation that relates the variables \( x \) and \( y \).
   (iii)(4pts) Now using optimization find the value of \( x \) and \( y \) that satisfy this problem. Justify your answer by classifying your critical point(s) using either the 1st or 2nd Derivative Test.
   (b)(12pts) Suppose we want to approximate the intersection points of \( y = x^2 \) and \( y = \cos(x) \). If we want to use Newton’s Method to do this then what would the formula for \( x_{n+1} \) be? (To get full points for this question you must provide the explicit formula for \( x_{n+1} \) in terms of \( x_n \), the generic formula for Newton’s Method is not sufficient. You do not need to approximate the point of intersection.)
   (c)(4pts) Suppose the tangent line to the curve \( y = f(x) \) at the point \((2, 5)\) has the equation \( y = 9 - 2x \). If Newton’s Method is used to locate a root of the equation \( f(x) = 1 \) and the initial approximation is \( x_1 = 2 \) then the second iteration is \( x_2 = \) see choices below (write answer in bluebook). (No justification necessary - Choose only one answer, copy down the entire answer in your bluebook.)

   (A) \( x_2 = \frac{2}{9} \)   (B) \( x_2 = 4 \)   (C) \( x_2 = 5 \)   (D) \( x_2 = \frac{9}{2} \)   (E) None of these.

PROBLEMS #3 & #4 ON THE OTHER SIDE
3. (24pts) The following problems are not related.

(a)(12pts) For what values of the constants $a$ and $b$ is $(1, 2)$ a point of inflection of the curve $y = ax^3 + bx^2$?

(b)(12pts) In your blue book clearly sketch the graph of a function $h(x)$ that satisfies all the following properties (label all extrema, inflection points and asymptotes):

- $h(x)$ is a polynomial and an odd function, $h(0) = 0$ and $h(1) = 2$,
- $\lim_{x \to -0.5^+} h(x) = -1.5$, $\lim_{x \to -\infty} h(x) = +\infty$, and $\lim_{x \to \infty} h(x) = -\infty$,
- The 1st derivative test for $h(x)$ is shown below:

Sign chart for $h'(x)$

\[ \begin{array}{ccc}
-1 & + & - \\
\downarrow & & \downarrow \\
-1 & & 1 \\
\end{array} \]

- $h''(x) < 0$ on $(-1/2, 0)$ and $(1/2, \infty)$.

4. (24pts) The following problems are not related.

(a)(10pts) Find the most general antiderivative of the function $f(x) = \sqrt{x} + \sec^2(x) + \pi^2$. Show all work.

(b)(10pts) Suppose the acceleration of an object at any time $t$ is given by $a(t) = 3t^2 - 4t$ m/s$^2$, $t \geq 0$. Find the velocity, $v(t)$, at any time $t$ if $v(1) = 1$ m/s. Show all work.

(c)(4pts) Which one of the choices below can be verified to be equivalent to $\int x \cos(x) \, dx$? (No justification necessary - Choose only one answer, copy down the entire answer in your bluebook.)

(A) $\frac{x^2}{2} \sin(x) + C$   (B) $\cos(x) + x \sin(x) + C$   (C) $x \sin(x) + C$   (D) $\frac{\cos^2(x)}{2} + C$   (E) $\sin(x) + C$