APPM 1340
Final Exam
Fall 2023

Name	
Instructor Richard McNamara	**Section** 150

This exam is worth 150 points and has **7 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End-of-Exam Checklist

1. If you finish the exam before 9:45 AM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.
2. If you finish the exam after 9:45 AM:
   - Please wait in your seat until 10:00 AM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors.

1. (32 pts) The position function of a particle is given by $s(t) = 4\sqrt{t} - t$ on the interval $1 \leq t \leq 16$, where position is in meters and time is in seconds.

   (a) Determine the particle's velocity function $v(t)$. Include the correct unit of measurement.

   (b) Determine the total distance traveled by the particle on the interval $1 \leq t \leq 16$. Include the correct unit of measurement.

(c) i. Verify that all hypotheses of the Mean Value Theorem are satisfied for the given position function $s(t) = 4\sqrt{t} - t$ on the interval $1 \leq t \leq 16$.

ii. Use the Mean Value Theorem to determine all time values c on the interval $1 \leq t \leq 16$, if any, for which the instantaneous velocity of the particle equals the average velocity of the particle on that interval. Include the correct unit of measurement.

2. (11 pts) Let v represent a person's walking speed, expressed in miles per hour, and let p represent the corresponding walking pace, expressed in minutes per mile. The pace can be expressed as the following function of speed:

$$p(v) = \frac{60}{v}, \quad v > 0$$

(a) Find the linearization $L(v)$ that approximates $p(v)$ near $v = 4$.

(b) Use your linearization from part (a) to estimate the walking pace of a person moving at 4.2 miles per hour. Include the correct unit of measurement. You must use linearization to earn credit.

3. (26 pts) Consider the function $f(x) = \sin x + \cos^2 x$ on the interval $[0, 2\pi/3]$.

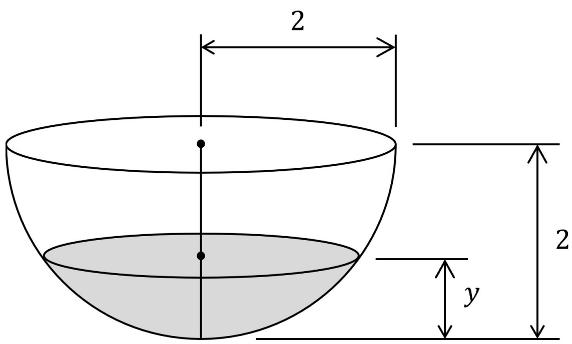
   (a) Identify all critical numbers of f on the specified interval.

   (b) Use the Closed Interval Method to find the absolute maximum and minimum values of f on the specified interval. Clearly identify all x values that are associated with the absolute maximum function value and all x values that are associated with the absolute minimum function value. Note that $\sqrt{3} \approx 1.7$.

4. (26 pts) Find an equation of each tangent line, if any, to the curve $2x^3 + 2y^2 = 5xy$ at $x = 1$.

5. (18 pts) Two bowls of identical size are both hemispheres of radius 2 ft. When such a bowl contains water having a depth of y ft, as depicted below, the corresponding volume of water in the bowl is given by the following function:

$$V = \pi y^2(2 - y/3)\ , \quad 0 \leq y \leq 2$$



(a) Suppose bowl A is being filled at a constant rate of $8\pi/9$ cubic ft per minute. How fast is the depth of the water increasing when the water is 1 ft deep? Simplify your answer fully and include the correct unit of measurement.

(b) Suppose bowl B is being filled in such a way that its water depth is increasing at a constant rate of $1/3$ ft per minute. How fast is the volume of the water increasing when the water is 1 ft deep? Simplify your answer fully and include the correct unit of measurement.

6. (15 pts) Determine $g'(x)$ for the function $g(x) = \sqrt{3x+1}$ by using the definition of derivative. You must obtain g' by evaluating an appropriate limit to earn credit.

7. (22 pts) Consider the function $h(x) = \dfrac{\sin x}{x(x - \pi/2)}$.

(a) Find the (x, y) coordinates of every removable discontinuity of $h(x)$, if any exist. Support your answer by evaluating the appropriate limit(s).

(b) Find the equation of every vertical asymptote of $y = h(x)$, if any exist. Support your answer by evaluating the appropriate limit(s).

END OF EXAM

Your Initials _________

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.