1. (30 pts) Determine $\frac{dy}{dx}$ for each of the following.

(a) $y = \frac{\sin x}{2x + 1}$

Solution:

$$\frac{dy}{dx} = \frac{(2x + 1) \cdot \frac{d}{dx}[\sin x] - \sin x \cdot \frac{d}{dx}[2x + 1]}{(2x + 1)^2} = \frac{(2x + 1) \cos x - 2 \sin x}{(2x + 1)^2}$$

(b) $x^3 - y^3 = 5xy$

Solution:

$$\frac{d}{dx}[x^3 - y^3] = \frac{d}{dx}[5xy]$$

$$3x^2 - 3y^2 y' = 5(xy' + y)$$

$$3x^2 - 5y = (5x + 3y^2)y'$$

$$y' = \frac{dy}{dx} = \frac{3x^2 - 5y}{5x + 3y^2}$$

(c) $y = 4 \cos^5 (2x)$

Solution:

$$y = 4 \cos^5 (2x) = 4[\cos (2x)]^5$$

$$\frac{dy}{dx} = (4)(5)[\cos (2x)]^4 \cdot \frac{d}{dx} \cos (2x) = 20 \cos^4 (2x) \left[ - \sin (2x) \cdot \frac{d}{dx}[2x] \right]$$

$$\frac{dy}{dx} = -40 \cos^4 (2x) \sin (2x)$$
2. (25 pts) The position value of a particle is given by \( s(t) = t^2 - 4t^{1.5} + 4t \), where \( t \geq 0 \) is in seconds and position is in feet. For each of the following, be sure to include the correct unit of measurement.

(a) Find the particle’s velocity function \( v(t) \).

\[
\text{Solution:} \quad v(t) = s'(t) = \frac{d}{dt} [t^2 - 4t^{1.5} + 4t] = 2t - (4)(1.5)t^{0.5} + 4 = 2t - 6t^{0.5} + 4 \text{ ft/s}
\]

(b) Determine the particle’s speed at \( t = \frac{9}{4} \) seconds.

\[
\text{Solution:} \quad \left| v \left( \frac{9}{4} \right) \right| = \left| (2) \left( \frac{9}{4} \right) - (6) \left( \frac{9}{4} \right)^{0.5} + 4 \right| = \left| \frac{9}{2} - (6) \left( \frac{3}{2} \right)^{0.5} + 4 \right| = \left| \frac{9}{2} - \frac{18}{2} + \frac{8}{2} \right| = \left| -\frac{1}{2} \right| = \frac{1}{2} \text{ ft/s}
\]

(c) Find the particle’s acceleration function \( a(t) \).

\[
\text{Solution:} \quad a(t) = v'(t) = \frac{d}{dt} [2t - 6t^{0.5} + 4] = 2 - (6)(0.5)t^{-0.5} = 2 - 3t^{-0.5} \text{ ft/s}^2
\]

(d) Find all values of \( t \geq 0 \) for which the particle’s acceleration is equal to 0.

\[
\text{Solution:} \quad 2 - 3t^{-0.5} = 0
\]
\[
2 = 3t^{-0.5}
\]
\[
t^{0.5} = \frac{3}{2}
\]
\[
t = \frac{9}{4} \text{ sec}
\]
3. (25 pts) Parts (a) and (b) are unrelated.

(a) Find the equations of the tangent and normal lines to the curve \( y = x^3 - 2x^2 + x + 10 \) at \( x = -1 \).

**Solution:**

\[
y(-1) = (-1)^3 - 2(-1)^2 + (-1) + 10 = -1 - 2 - 1 + 10 = 6
\]

The point of tangency is \((-1, 6)\).

\[
y'(x) = 3x^2 - 4x + 1
\]

\[
y'(-1) = (3)(-1)^2 - (4)(-1) + 1 = 3 + 4 + 1 = 8
\]

The slope of the tangent line at \((-1, 6)\) is 8.

The equation of the tangent line is \( y - 6 = 8(x - (-1)) \) or \( y - 6 = 8(x + 1) \)

The slope of the normal line is \(-\frac{1}{\text{slope of tangent line}} = -\frac{1}{8}\).

The equation of the normal line is \( y - 6 = -\frac{1}{8}(x + 1) \)

(b) Find all values of \( x \) on the interval \([0, \pi]\) for which the curve \( y = \tan x - 4x \) has a horizontal tangent line.

**Solution:**

\[
y'(x) = \sec^2 x - 4 = 0
\]

\[
\sec^2 x = 4
\]

\[
\sec x = \pm 2
\]

\[
\cos x = \pm \frac{1}{2}
\]

The solutions on the interval \([0, \pi]\) are \( x = \frac{\pi}{3}, \frac{2\pi}{3} \)
4. (20 pts) Parts (a) and (b) are unrelated.

(a) Determine $f'(x)$ for the function $f(x) = \sqrt{x + 1}$ by using the definition of derivative. (You must obtain $f'$ by evaluating the appropriate limit to earn credit.)

Solution:

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x + h + 1} - \sqrt{x + 1}}{h}
\]

\[
= \lim_{h \to 0} \frac{(x + h + 1) - (x + 1)}{h} \cdot \frac{\sqrt{x + h + 1} + \sqrt{x + 1}}{\sqrt{x + h + 1} + \sqrt{x + 1}}
\]

\[
= \lim_{h \to 0} \frac{h}{h(\sqrt{x + h + 1} + \sqrt{x + 1})} = \lim_{h \to 0} \frac{1}{\sqrt{x + h + 1} + \sqrt{x + 1}}
\]

\[
= \frac{1}{2\sqrt{x + 1}}
\]

(b) $\lim_{x \to 1} \frac{x^8 + 2x^5 - 3}{x - 1}$ represents the derivative of a certain function $f$ at a certain number $a$.

i. Identify $f$ and $a$.

Solution:

The definition of a derivative states that $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

Since the given expression represents $f'(a)$ for some $f$ and $a$, we have

\[
f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{x^8 + 2x^5 - 3}{x - 1}
\]

Based on the structure of the terms in the preceding equation, we can hypothesize that $a = 1$ and $f(x) = x^8 + 2x^5$, in which case $f(a)$ would have to equal 3.

For $a = 1$ and $f(x) = x^8 + 2x^5$, we have $f(1) = 1^8 + (2)(1^5) = 3$, which confirms our hypothesis.

Therefore, $f(x) = x^8 + 2x^5$ and $a = 1$.

ii. Use $f'(a)$ to evaluate the given limit.

Solution:

Based on the work in part (i), $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{x^8 + 2x^5 - 3}{x - 1} = f'(1)$.

\[
f'(x) = 8x^7 + 10x^4 \implies f'(1) = (8)(1^7) + (10)(1^4) = 18 \implies \lim_{x \to 1} \frac{x^8 + 2x^5 - 3}{x - 1} = 18
\]