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1. (30 pts) Determine $\frac{dy}{dx}$ for each of the following.

   (a) $y = \frac{\sin x}{2x+1}$

   Solution:

   $$\frac{dy}{dx} = \frac{(2x+1)\cdot\frac{d}{dx}[\sin x] - \sin x \cdot \frac{d}{dx}[2x+1]}{(2x+1)^2} = \boxed{\frac{(2x+1)\cos x - 2\sin x}{(2x+1)^2}}$$

   (b) $x^3 - y^3 = 5xy$

   Solution:

   $$\frac{d}{dx}[x^3 - y^3] = \frac{d}{dx}[5xy]$$

   $$3x^2 - 3y^2y' = 5(xy' + y)$$

   $$3x^2 - 5y = (5x + 3y^2)y'$$

   $$y' = \frac{dy}{dx} = \boxed{\frac{3x^2 - 5y}{5x + 3y^2}}$$

   (c) $y = 4\cos^5(2x)$

   Solution:

   $$y = 4\cos^5(2x) = 4[\cos(2x)]^5$$

   $$\frac{dy}{dx} = (4)(5)[\cos(2x)]^4 \cdot \frac{d}{dx}[\cos(2x)] = 20\cos^4(2x)\left[-\sin(2x)\cdot\frac{d}{dx}[2x]\right]$$

   $$\frac{dy}{dx} = \boxed{-40\cos^4(2x)\sin(2x)}$$

2. (25 pts) The position value of a particle is given by $s(t) = t^2 - 4t^{1.5} + 4t$, where $t \geq 0$ is in seconds and position is in feet. For each of the following, be sure to include the correct unit of measurement.

(a) Find the particle's velocity function $v(t)$.

Solution:

$$v(t) = s'(t) = \frac{d}{dt}\left[t^2 - 4t^{1.5} + 4t\right] = 2t - (4)(1.5)t^{0.5} + 4 = \boxed{2t - 6t^{0.5} + 4 \text{ ft/s}}$$

(b) Determine the particle's speed at $t = \dfrac{9}{4}$ seconds.

Solution:

$$\left|v\left(\frac{9}{4}\right)\right| = \left|(2)\left(\frac{9}{4}\right) - (6)\left(\frac{9}{4}\right)^{0.5} + 4\right| = \left|\frac{9}{2} - (6)\left(\frac{3}{2}\right) + 4\right| = \left|\frac{9}{2} - \frac{18}{2} + \frac{8}{2}\right| = \left|-\frac{1}{2}\right| = \boxed{\frac{1}{2} \text{ ft/s}}$$

(c) Find the particle's acceleration function $a(t)$.

Solution:

$$a(t) = v'(t) = \frac{d}{dt}\left[2t - 6t^{0.5} + 4\right] = 2 - (6)(0.5)t^{-0.5} = \boxed{2 - 3t^{-0.5} \text{ ft/s}^2}$$

(d) Find all values of $t \geq 0$ for which the particle's acceleration is equal to 0.

Solution:

$$2 - 3t^{-0.5} = 0$$

$$2 = 3t^{-0.5}$$

$$t^{0.5} = \frac{3}{2}$$

$$t = \boxed{\frac{9}{4} \text{ sec}}$$

3. (25 pts) Parts (a) and (b) are unrelated.

(a) Find the equations of the tangent and normal lines to the curve $y = x^3 - 2x^2 + x + 10$ at $x = -1$.

Solution:

$y(-1) = (-1)^3 - 2(-1)^2 + (-1) + 10 = -1 - 2 - 1 + 10 = 6$

The point of tangency is $(-1, 6)$.

$y'(x) = 3x^2 - 4x + 1$

$y'(-1) = (3)(-1)^2 - (4)(-1) + 1 = 3 + 4 + 1 = 8$

The slope of the tangent line at $(-1, 6)$ is 8.

The equation of the tangent line is $y - 6 = 8(x - (-1)) = \boxed{y - 6 = 8(x + 1)}$

The slope of the normal line is $-\dfrac{1}{\text{slope of tangent line}} = -\dfrac{1}{8}$.

The equation of the normal line is $\boxed{y - 6 = -\dfrac{1}{8}(x + 1)}$

(b) Find all values of x on the interval $[0, \pi]$ for which the curve $y = \tan x - 4x$ has a horizontal tangent line.

Solution:

$y'(x) = \sec^2 x - 4 = 0$

$\sec^2 x = 4$

$\sec x = \pm 2$

$\cos x = \pm \dfrac{1}{2}$

The solutions on the interval $[0, \pi]$ are $x = \boxed{\dfrac{\pi}{3}, \dfrac{2\pi}{3}}$

4. (20 pts) Parts (a) and (b) are unrelated.

   (a) Determine $f'(x)$ for the function $f(x) = \sqrt{x+1}$ by using the **definition of derivative**. (You must obtain f' by evaluating the appropriate limit to earn credit.)

   Solution:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \lim_{h\to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}} = \boxed{\frac{1}{2\sqrt{x+1}}}$$

   (b) $\displaystyle\lim_{x\to 1} \frac{x^8 + 2x^5 - 3}{x-1}$ represents the derivative of a certain function f at a certain number a.

      i. Identify f and a.

      Solution:

      The definition of a derivative states that $f'(a) = \displaystyle\lim_{x\to a} \frac{f(x) - f(a)}{x-a}$.

      Since the given expression represents $f'(a)$ for some f and a, we have

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a} = \lim_{x\to 1} \frac{x^8 + 2x^5 - 3}{x-1}$$

      Based on the structure of the terms in the preceding equation, we can hypothesize that $a = 1$ and $f(x) = x^8 + 2x^5$, in which case $f(a)$ would have to equal 3.

      For $a = 1$ and $f(x) = x^8 + 2x^5$, we have $f(1) = 1^8 + (2)(1^5) = 3$, which confirms our hypothesis.

      Therefore, $\boxed{f(x) = x^8 + 2x^5}$ and $\boxed{a = 1}$.

      ii. Use $f'(a)$ to evaluate the given limit.

      Solution:

      Based on the work in part (i), $\displaystyle\lim_{x\to a} \frac{f(x) - f(a)}{x-a} = \lim_{x\to 1} \frac{x^8 + 2x^5 - 3}{x-1} = f'(1)$.

$$f'(x) = 8x^7 + 10x^4 \;\Rightarrow\; f'(1) = (8)(1^7) + (10)(1^4) = 18 \;\Rightarrow\; \lim_{x\to 1} \frac{x^8 + 2x^5 - 3}{x-1} = \boxed{18}$$