1. (28 pts) The following problems are not related.

   (a) Express the following as a polynomial: $(\sqrt{x-3}+2)(\sqrt{x-3}-2)$

   Solution: Using the FOIL method produces a difference of squares.

   $(\sqrt{x-3}+2)(\sqrt{x-3}-2) = (\sqrt{x-3})^2 - 2^2 = (x-3) - 4 = \boxed{x-7}$

   (b) Fully simplify: $\sqrt{a^2b^7}$

   Solution:

   $\sqrt{a^2b^7} = \sqrt{a^2}\sqrt{b^7} = \boxed{|a|b^3\sqrt{b}}$

   (c) Fully simplify: $\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$

   Solution:

   $$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{1}{h}\left[\frac{1}{x+h} - \frac{1}{x}\right] = \frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right] = \frac{-h}{hx(x+h)} = \boxed{-\frac{1}{x(x+h)}}$$

   (d) Solve: $x^{7/3} - 15x^{1/3} = 2x^{4/3}$

   Solution:

   $x^{7/3} - 15x^{1/3} = 2x^{4/3} \quad \Rightarrow \quad x^{7/3} - 2x^{4/3} - 15x^{1/3} = 0 \quad \Rightarrow \quad x^{1/3}(x^2 - 2x - 15) = 0$

   The Zero Factor Theorem indicates that $x^{1/3} = 0$ or $x^2 - 2x - 15 = 0$.

   $x^{1/3} = 0 \quad \Rightarrow \quad x = 0$

   $x^2 - 2x - 15 = (x-5)(x+3) = 0 \quad \Rightarrow \quad x = -3, 5$

   $\boxed{x = -3, 0, 5}$

2. (22 pts) For the following, let point A be $(2, 1)$ and let point B be $(-6, 7)$:

(a) Find the distance between points A and B.

Solution: Apply the distance formula.

$$D = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6-2)^2 + (7-1)^2} = \sqrt{(-8)^2 + 6^2}$$

$$D = \sqrt{100} = \boxed{10}$$

(b) Find a point-slope equation of the line passing through points A and B.

Solution:

The point-slope form of the equation of the line passing through a point (x_1, y_1) is $y - y_1 = m(x - x_1)$. Since the line of interest passes through points A and B, the coordinates of either point can be used as (x_1, y_1).

The parameter m, which represents the slope of the line, is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

It does not matter which point is selected to be (x_1, y_1) and which is selected to be (x_2, y_2). The two possibilities produce the same slope value:

$$m = \frac{7-1}{-6-2} = \frac{1-7}{2-(-6)} = -\frac{3}{4}$$

Similarly, it does not matter which point is selected to be (x_1, y_1) in the equation of the line:

Using point A: $x_1 = 2,\ y_1 = 1$: $\boxed{y - 1 = -\frac{3}{4}(x-2)}$

Using point B: $x_1 = -6,\ y_1 = 7$: $y - 7 = -\frac{3}{4}(x - (-6)) \quad \Rightarrow \quad \boxed{y - 7 = -\frac{3}{4}(x+6)}$

Both of the preceding results are valid point-slope representations of the line passing through points A and B.

(c) Find the midpoint of the line segment connecting points A and B.

Solution:

The general expression for the midpoint of the line segment connecting the points (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Substituting the coordinate values of points A and B leads to the following midpoint of the connecting line segment:

$$\left(\frac{2-6}{2}, \frac{1+7}{2}\right) = \boxed{(-2, 4)}$$

(d) Find the equation of the line that is perpendicular to the line in part (b) and passes through the origin.

Solution:

The slopes of two perpendicular lines are negative reciprocals of each other. Since the slope of the line through points A and B is $-3/4$, the slope of a line that is perpendicular to the original line is $4/3$.

Since the new line is said to pass through the origin, the point-slope form and the slope-intercept form of that line will be the same:

Point-slope form: $y - 0 = \frac{4}{3}(x - 0)$

Slope-intercept form: $y = \frac{4}{3}x + 0$

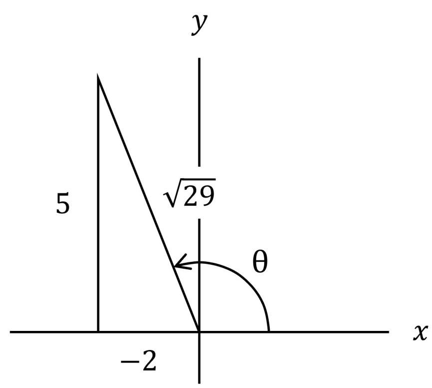
Therefore, the equation of the perpendicular line is $\boxed{y = \frac{4}{3}x}$

3. (22 pts) The following problems are not related.

(a) If θ is on the interval $\left[\frac{\pi}{2}, \pi\right]$ and $\tan\theta = -\frac{5}{2}$, find the value of $\csc\theta$.

Solution:

$\tan\theta = -\frac{5}{2}$ and $\frac{\pi}{2} \le \theta \le \pi$ together imply the following orientation of θ:



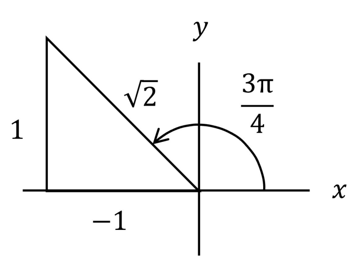
The hypotenuse in the preceding figure was determined from the Pythagorean Theorem: $\sqrt{(-2)^2 + 5^2} = \sqrt{29}$.

It follows from the figure that $\csc\theta = \boxed{\frac{\sqrt{29}}{5}}$

(b) Evaluate $\cos\left(\frac{3\pi}{4}\right)$.

Solution:

Since $\frac{\pi}{2} < \frac{3\pi}{4} < \pi$, the angle $\frac{3\pi}{4}$ lies in Quadrant II, as drawn in the following figure.



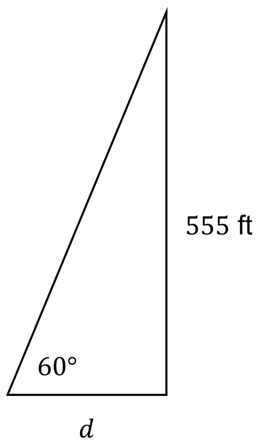
The reference angle is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$, which is a $45°$ angle in the special $45° - 45° - 90°$ right triangle. The dimensions of such a triangle are proportional to 1, 1, and $\sqrt{2}$, which leads to the set of dimensions displayed in the figure.

It follows from the figure that $\cos\left(\frac{3\pi}{4}\right) = \boxed{-\frac{1}{\sqrt{2}}}$

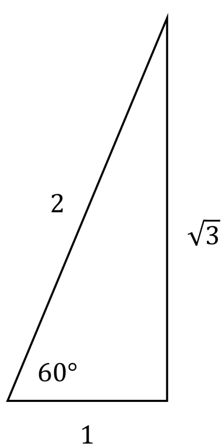
(c) The height of a building is known to be 555 feet. A person standing a certain distance away measures the angle from their feet to the top of the building to be $60°$. How far away is the person from the building? Express your answer in exact form and include the proper unit of measurement.

Solution:

The situation is depicted in the following figure:



The triangle in the preceding figure is the special $30° - 60° - 90°$ right triangle, which is depicted below:



Since the two triangles depicted above are similar triangles, the following proportion of side lengths applies, which is also equal to $\tan 60°$:

$$\tan 60° = \frac{555}{d} = \sqrt{3}$$

Solving for d and applying the appropriate measurement unit of feet results in $\boxed{d = \frac{555}{\sqrt{3}} \text{ ft}}$

4. (28 pts) The following problems are not related.

(a) Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin(2x) = \cos x$.

Solution:

The double-angle formula $\sin 2x = 2\sin x \cos x$ can be substituted for the left-hand expression in the given equation:

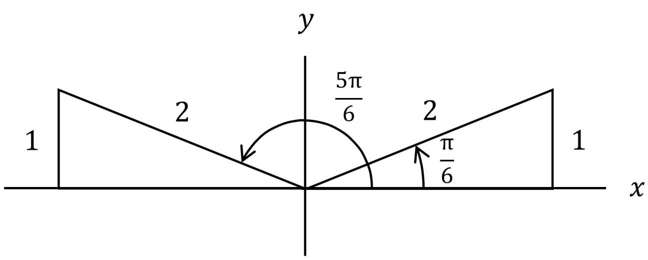
$$2\sin x\cos x = \cos x \quad \Rightarrow \quad 2\sin x\cos x - \cos x = 0 \quad \Rightarrow \quad \cos x(2\sin x - 1) = 0$$

The Zero Factor Theorem indicates that $\cos x = 0$ or $2\sin x - 1 = 0$.

$$\cos x = 0 \quad \Rightarrow \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2\sin x - 1 = 0 \quad \Rightarrow \quad \sin x = \frac{1}{2}$$

The following figure can assist in evaluating $\sin x = \dfrac{1}{2}$:



Both triangles in the preceding figure are associated with an angle whose sine is $1/2$. The leg of length 1 and the hypotenuse of length 2 together imply that both triangles are special $30^\circ - 60^\circ - 90^\circ$ right triangles. In such a triangle, the angle opposite the leg of length 1 is 30°, which is $\pi/6$ radians. Therefore, the two solutions to the equation $\sin x = 1/2$ are:

$$x = \frac{\pi}{6}, \;\; \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\boxed{x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

(b) What is the radius of a circular sector having a central angle of $40°$ and an area of 4π?

Solution:

The formula for the area of a circular sector is $A = \frac{1}{2}\theta r^2$.

In this problem, the central angle θ is $40°$, which is $(40°)\left(\frac{\pi \text{ rad}}{180°}\right) = \frac{2\pi}{9}$ radians.

Substituting $\theta = \frac{2\pi}{9}$ and $A = 4\pi$ into the equation for area yields:

$$4\pi = \frac{1}{2} \cdot \frac{2\pi}{9} \cdot r^2 \quad \Rightarrow \quad r^2 = 36 \quad \Rightarrow \quad \boxed{r = 6}$$

(c) Is the function $f(x) = x^7 + 3x^3 - 1$ odd, even, or neither? Justify your answer by using the definition of odd and/or even functions.

Solution:

$$f(-x) = (-x)^7 + (3)(-x)^3 - 1 = -x^7 - 3x^3 - 1$$

$f(-x) = -x^7 - 3x^3 - 1 \neq f(x) \quad \Rightarrow$ f is not an even function

$f(-x) = -x^7 - 3x^3 - 1 \neq -f(x) \quad \Rightarrow$ f is not an odd function

Therefore, $\boxed{f \text{ is neither even nor odd}}$