APPM 1340 **Final (150pts)** **Fall 2021**

INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1340/Final**, (3) **lecture number/instructor name** and (4) **FALL 2021** on the front of your bluebook. Do all problems. **Start each problem on a new page.** $\boxed{\textbf{Box}}$ **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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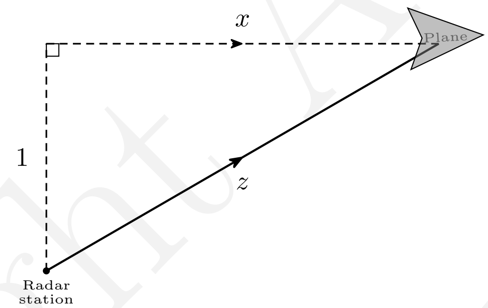
1. (35pts) The following problems are not related.

   (a)(16pts) Consider the following problem: *A plane flying horizontally at an altitude of* $1mi$ *and a speed of* $500mi/h$ *passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is* $2mi$ *away from the station.* **Now answer the following questions:** (i)(4pts) Write down the information that is given in this problem (use the notation established in the diagram below). (ii)(4pts) Write down what you are trying to find in this problem. (iii)(8pts) Solve the problem. Simplify your answer.



Problem 1: Figure for problem 1(a). The plane has a constant altitude of 1 mile.

   (b)(16pts) Find the equation of the *tangent line* to $y = \sqrt{x}$ at the point $(1, 1)$. Simplify your answer.

   (c)(3pts) Which choice below is the correct *derivative* of $f(x) = \dfrac{x^2-2}{2x+1}$? (**No justification necessary** - *Choose only one answer, copy down the entire answer.*)

   (A) $f'(x) = \dfrac{2x}{(2x+1)^2}$ (B) $f'(x) = \dfrac{3x^2-3}{(2x+1)^2}$ (C) $f'(x) = \dfrac{2x^2+2x+4}{2x^2+4x+1}$ (D) $f'(x) = \dfrac{2(x^2+x+2)}{(2x+1)^2}$ (E) $f'(x) = \dfrac{2x^2+2x+4}{(x+2)^2}$

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2. (34pts) Start this problem on a **new** page. The following problems are not related.

   (a)(17pts) Suppose r represents the radius of a circular disk. (i)(8pts) If $A(r) = \pi r^2$ find dA, the *differential* of A. (ii)(9pts) Suppose the radius of the circle was originally found to be 10 cm but expands to 10.2 cm, use differentials to estimate the change in the area, ΔA.

   (b)(17pts) Find the *absolute minimum* and *absolute maximum values* of $f(x) = (x^2-1)^3$ on the interval $[-1, 2]$. Give your answer in the form (x, y). Show all work, justify your answers and clearly label your answers.

PROBLEMS #3, #4 & #5 ON THE OTHER SIDE

3. (34pts) Start this problem on a **new** page. The following problems are not related.

(a)(17pts) Use the *Squeeze Theorem* to evaluate the following limit: $\lim\limits_{x \to 0^+} \sqrt{x}\cos^2\left(\frac{1}{x}\right)$. Show all work, explain.

(b)(17pts) Suppose $y = f(x)$, use *implicit differentiation* to find y' if $y\cos(x) = x^2 + y^2$.

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4. (35pts) Start this problem on a **new** page. The following problems are not related.

(a)(16pts) Suppose $g(x) = \begin{cases} x^2 + x, & \text{if } x < 0, \\ 1 - \cos(x), & \text{if } x = 0, \\ \sin(x), & \text{if } x > 0. \end{cases}$ (i)(8pts) Find the $\lim\limits_{x \to 0} g(x)$. (ii)(8pts) Show that $g(x)$ is continuous at $x = 0$. Be sure to show that all the conditions of continuity have been satisfied.

(b)(16pts) (i)(8pts) Write down the *piecewise* definition of the function $f(x) = 1 + |x^2 - 4|$.

(ii)(8pts) Find the derivative of $f(x) = 1 + |x^2 - 4|$.

(c)(3pts) The function $h(x) = \dfrac{3x+1}{\sqrt[3]{5 + 8x^3}}$ has a *horizontal asymptote* at which choice below? (**No justification necessary** - *Choose only <u>one</u> answer, copy down the entire answer.*)

(A) $y=0$ (B) $y=1.5$ (C) $y=0$ and $y=3/2$ (D) $y=-3/2$ and $y=3/2$ (E) None of these

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5. (12pts) Answer either **ALWAYS TRUE** or **FALSE**. You do <u>NOT</u> need to justify your answer. (*Don't just write down "A.T." or "F", completely write out the words "ALWAYS TRUE" or "FALSE" depending on your answer.*)

(a)(3pts) The Mean Value Theorem applies to $h(x) = -\dfrac{1}{x}$ on $[-3, -\frac{1}{2}]$ and the value of $c \in (-3, -\frac{1}{2})$ that satisfies the Mean Value Theorem is $c = -\frac{2}{3}$.

(b)(3pts) Suppose $f(x)$ is *continuous* for all x, then $f(x)$ is *differentiable* at $x = a$ if $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists.

(c)(3pts) Suppose the position function of a particle (at time $t \geq 0$ in seconds) is given by $s(t) = t^2 - t$ meters, then the *total distance* traveled during the time period $0 \leq t \leq 1$ by the particle is 0.5 meters.

(d)(3pts) The function $f(x) = \dfrac{x-2}{x^3 - 2x^2}$ has a *vertical asymptote* at $x = 0$ and a *jump discontinuity* at $x = 2$.

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