1. (28pts) The following problems are not related. Show all work. Simplify your answers.

(a)(20pts)(i)(10pts) Assume x is a positive real number and find the product: $(x^{1/2} - x^{3/2})^2$

(ii)(10pts) Assume $p \in \mathbb{R}$ and $p > 0$ and perform the indicated operation (write your answer with positive exponents only): $(p+4)^{-3/2} + (p+4)^{1/2}$

(b)(8pts) Use the Quadratic Formula to solve the equation: $x^2 - x = 1$

Solution: (a)(i)(10pts) Here we have

$$(x^{1/2} - x^{3/2})^2 = (x^{1/2})^2 - 2x^{1/2} \cdot x^{3/2} + (x^{3/2})^2 = x - 2x^{4/2} + x^3 = \boxed{x - 2x^2 + x^3.}$$

(a)(ii)(10pts) Combining the fractions gives

$$\begin{aligned}(p+4)^{-3/2} + (p+4)^{1/2} &= \frac{1}{(p+4)^{3/2}} + (p+4)^{1/2} \\ &= \frac{1}{(p+4)^{3/2}} + \frac{(p+4)^{1/2} \cdot (p+4)^{3/2}}{(p+4)^{3/2}} \\ &= \frac{1 + \left(p+4\right)^{4/2}}{(p+4)^{3/2}} = \frac{1 + \left(p+4\right)^2}{(p+4)^{3/2}} = \frac{1 + \left(p^2 + 8p + 16\right)}{(p+4)^{3/2}} = \boxed{\frac{p^2 + 8p + 17}{(p+4)^{3/2}}.}\end{aligned}$$

(b)(8pts) First we make this a root finding problem and then use the Quadratic Formula,

$$x^2 - x = 1 \;\Rightarrow\; x^2 - x - 1 = 0 \;\Rightarrow\; a = 1,\, b = -1,\, c = -1 \;\Rightarrow\; x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \boxed{\frac{1 \pm \sqrt{5}}{2}.}$$

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2. (24pts) The following problems are not related. Show all work. Simplify your answers

(a)(10pts) Assuming all the variables are positive, simplify the rational expression: $\dfrac{p^{-1} + q^{-1}}{(pq)^{-1}}$

(b)(10pts) Solve the equation by factoring the polynomial: $18x^2 + 9x - 2 = 0$

(c)(4pts) Which choice below is equivalent to $\sqrt{\dfrac{g^3 h^5}{r^3}}$ if all the variables are positive? **Choose only <u>one</u> answer.** *No justification necessary, copy down the entire answer. If you do not copy down the entire answer, points will be deducted.*

(A) $\dfrac{g^6 h^{10}}{r^6}$ (B) $\dfrac{g^{3.2} h^{5.2}}{r^{3.2}}$ (C) $\dfrac{gh^2\sqrt{ghr}}{r^2}$ (D) $\dfrac{gh^2\sqrt{gh}}{r}$ (E) None of these

Solution: (a)(10pts) Here we have

$$\frac{p^{-1}+q^{-1}}{(pq)^{-1}} = \frac{\frac{1}{p}+\frac{1}{q}}{\frac{1}{pq}} = \frac{\frac{q+p}{pq}}{\frac{1}{pq}} = \frac{q+p}{pq}\cdot\frac{pq}{1} = \boxed{q+p.}$$

(b)(10pts) By trial and error with the factors of 18 and 2 we have the factorization

$$18x^2+9x-2=(6x-1)(3x+2) \;\Rightarrow\; (6x-1)(3x+2)=0 \;\Rightarrow\; \boxed{x=-\frac{2}{3},\,\frac{1}{6}.}$$

(c)(4pts) $\boxed{\text{Choice C.}}$ Note that

$$\sqrt{\frac{g^3h^5}{r^3}} = \sqrt{\frac{g^2\cdot g\cdot h^4\cdot h}{r^2\cdot r}} = \frac{gh^2\sqrt{gh}}{r\sqrt{r}} = \frac{gh^2\sqrt{gh}}{r\sqrt{r}}\cdot\frac{\sqrt{r}}{\sqrt{r}} = \frac{gh^2\sqrt{ghr}}{r^2} \Rightarrow \text{(C)}.$$

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3. (20pts) The following problems are not related. Show all work. Simplify your answers.

(a)(10pts) Find all solutions of the equation $\sin^2(\theta)\cos(2\theta)=\cos(2\theta)$ that are in the interval $0\le\theta<2\pi$.

(b)(10pts) Write down the *piecewise* definition of the function $f(x)=1+|x^2-4|$.

Solution:

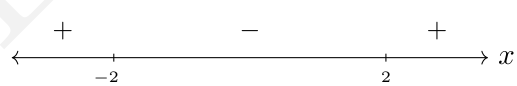
(a)(10pts) We make this a root finding problem and use the Zero Factor property,

$$\sin^2(\theta)\cos(2\theta)=\cos(2\theta) \;\Rightarrow\; \sin^2(\theta)\cos(2\theta)-\cos(2\theta)=0 \;\Rightarrow\; \cos(2\theta)\left[\sin^2(\theta)-1\right]=0$$
$$\Rightarrow\; \cos(2\theta)=0 \;\text{ and }\; \sin(\theta)=\pm1$$

and now note that

$$\cos(2\theta)=0 \;\Rightarrow\; 2\theta=\frac{\pi}{2},\frac{3\pi}{2} \;\Rightarrow\; \theta=\frac{\pi}{4},\frac{3\pi}{4} \;\text{ and }\; \sin(\theta)=\pm1 \;\Rightarrow\; \theta=\frac{\pi}{2},\frac{3\pi}{2} \;\Rightarrow\; \boxed{\theta=\frac{\pi}{4},\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2}.}$$

(b)(10pts) Note that $x^2-4=(x-2)(x+2)$ and so we can check the sign of $f(x)$ with a number line



thus we have

$$f(x)=1+|x^2-4| = \begin{cases} 1+(x^2-4), & \text{if } x\le-2 \text{ or } x\ge2, \\ 1-(x^2-4), & \text{if } -2<x<2, \end{cases} = \boxed{\begin{cases} x^2-3, & \text{if } x\le-2 \text{ or } x\ge2, \\ 5-x^2, & \text{if } -2<x<2. \end{cases}}$$

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4. (28pts) The following problems are not related. Show all work. Simplify your answers

(a)(12pts) Use the formula $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ and the fact that $75° = 30° + 45°$ to find the exact value of $\cos(75°)$.

(b)(12pts) Suppose $\frac{\pi}{2} \leq \theta \leq \pi$, find $\tan(\theta)$ given that $\sin(\theta) = \dfrac{1}{3}$.

(c)(4pts) If we solve the equation $1 + x + xy = y - xy$ for variable x then which choice below is equal to x? **Choose only <u>one</u> answer.** *No justification necessary, copy down the entire answer. If you do not copy down the entire answer, points will be deducted.*

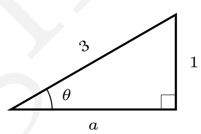
(A) $x = y - 1$ (B) $x = \dfrac{y-1}{2y+1}$ (C) $x = \dfrac{y}{1+2y}$ (D) $x = \dfrac{2y-1}{1+y}$ (E) None of these

Solution: (a)(12pts) Using the given formula, we have

$$\cos(75°) = \cos(30° + 45°) = \cos(30°)\cos(45°) - \sin(30°)\sin(45°) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}.$$

(b)(12pts) If we build the triangle from the given information we have

$\sin(\theta) = \dfrac{1}{3} = \dfrac{\text{opp}}{\text{hyp}} \Rightarrow$



$\Rightarrow \quad a = \pm\sqrt{3^2 - 1^2} = \pm\sqrt{8} = \pm 2\sqrt{2}$

and, if $\frac{\pi}{2} \leq \theta \leq \pi$ then $\tan(\theta) < 0$, so we have

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = -\frac{1}{\sqrt{8}} = \boxed{-\frac{1}{2\sqrt{2}}.}$$

(c)(4pts) $\boxed{\text{Choice B.}}$ Here we have

$$1 + x + xy = y - xy \Rightarrow x + 2xy = y - 1 \Rightarrow x(1+2y) = y - 1 \Rightarrow x = \frac{y-1}{1+2y} \Rightarrow \text{(B)}.$$