INSTRUCTIONS: Books, notes, and electronic devices are **not** permitted. Write **your full name** on every piece of paper that will be uploaded to gradescope. Do all problems. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work. Only use techniques from sections 1.1-3.2.**

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1. (25pts) Short answer. No justification required.

(a) Find $\sin(x)$, given $\cos(x) = 5/13$

(b) Evaluate

$$\lim_{x\to 0} \frac{sin(5x)}{3x}$$

(c) Evaluate

$$\lim_{t\to\infty} \frac{5t^2 - 6t + 25}{1340t^2 - 1340t}$$

(d) True or False: If a function is continuous it must be differentiable.

(e) True of False: It is possible a function is continuous on $[a, b]$ where $a < b$ and differentiable on (a, b) and has these three properties: $f(a) = 0$, $f(b) = 0$, and $f'(x) > 0$ for all x.

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2. (25pts) Justify all work.

(a) If $\frac{d}{dx}[\ln x] = \frac{1}{x}$, evaluate

$$\frac{d}{dx}[\ln x^2]$$

(b) Find the derivative of

$$f(x) = \tan(\cos(x^2))$$

(c) Find dy/dx for

$$x^3y^2 = 6\sin(y) + 5$$

(d) Find the tangent line to $y^2 + x^2 = 2$ at the point $(1, 1)$.

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3. (25pts) Show all work.

(a) Suppose two resistors are connected in parallel with resistances R_1 and R_2 measured in Ohms (Ω). The total resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose R_1 is increasing at a rate of $1/4\ \Omega/s$ and R_2 is increasing at a rate of $1/16\ \Omega/s$. At what rate is R changing when $R_1 = 1/2\ \Omega$ and $R_2 = 1/4\ \Omega$.

(b) Car A is traveling North at a rate of 60 mi/hr, car B is traveling West at a rate of 45 mi/hr. The cars are on straight roads approaching an intersection point. At what rate is the distance between the cars changing when Car A is 3 miles from the intersection and Car B is 4 miles from the intersection.

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4. (25pts) Show all work.

(a) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ about $a = 0$. Use it to approximate the square root of 9/10.

(b) You are eating a cylindrical stack of pancakes that has a radius and height of 5 cm. You want to cover your whole stack in a thin layer of syrup that is 1/10 cm thick. Use differentials to estimate the change in volume of your stack of pancakes after adding the layer of syrup. Use that the volume of a cylinder is $V = \pi r^2 h$.

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5. (25pts) Show all work.

(a) Find the absolute max and min of $f(x) = \frac{4}{3}x^3 - x$ on the interval $[-3, 0]$.

(b) Draw a graph corresponding to a function $g(x)$ defined on the interval $[0, 4]$ with the following features:

- g is continuous on $[0, 4]$
- g is differentiable on $(0, 4)$
- g(1) is a local max
- g(2) is an absolute min
- g(3) is a local max
- g(4) is an absolute max

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6. (25pts) Show all work.

(a) State the Mean Value Theorem.

(b) Suppose we know that $f(x)$ is continuous on the interval $[-7, 0]$ and differentiable on the interval $(-7, 0)$, that $f(-7) = -3$ and that $f'(x) \leq 2$ for all x. What is the largest possible value for $f(0)$.