

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write **your full name** on every piece of paper that will be uploaded to gradescope. Do all problems. **Start each problem on a new page.** **Box** your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work. Only use techniques from sections 1.1-2.6.**

1. (24pts) Short answer. No justification required.

(a) Evaluate

$$\lim_{x \rightarrow \pi/6} \frac{\sin(x) - 1/2}{x - \pi/6}$$

(b) Find the first, second, third, fourth, and 1340th derivative of

$$f(x) = x^2 + \cos(x)$$

(c) Find $f'(4)$ if

$$f(x) = \frac{\sqrt{g(x)}}{x}, \quad g(4) = 4, \quad g'(4) = 2.$$

2. (24pts) Justify all work.

(a) Use the definition of tangent,

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

and the quotient rule to show that

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

(b) Find the derivative of

$$y = \tan(x^3 + 1)$$

3. (26pts) Justify all work.

(a) If $y = A \sin(x) + B \cos(x)$, find constants A and B so that

$$y'' + y' - 2y = \cos(x)$$

(b) Find a tangent line of the curve $y = 5x^2 + 1$ that is perpendicular to the line $y = \frac{1}{5}x + 2$.

Hint: Two lines $y = m_1x + b_1$ and $y = m_2x + b_2$ are perpendicular if their slopes are negative reciprocals, that is if $m_1 = \frac{-1}{m_2}$ or $m_2 = \frac{-1}{m_1}$.

4. (26pts) Justify all work.

(a) Find dy/dx for

$$\sin(xy) = \frac{x}{y}.$$

Do not leave negative exponents in your final answer.

(b) A particle's position is given by

$$s(t) = 7t^2 - 28t + 15.$$

Answer the following questions about this particle:

- Find equations of velocity and acceleration for the particle.
- When is the particle at rest?
- Set up, but do not evaluate, an expression for the total distance traveled by the particle between times $t = 0$ and $t = 3$.