1. (28pts) The following problems are not related. Show all work.

(a)(20pts)(i)(10pts) Express as a polynomial: $(3x-1)(x+2)+7x(x+1)$.

(ii)(10pts) Simplify the expression: $(3x+2)^{1/3}(2)(4x-5)(4)+(4x-5)^2\left(\frac{1}{3}\right)(3x+2)^{-2/3}(3)$.

(b)(8pts) Use the *quadratic formula* to solve the equation: $1+3x^2=-5x$.

Solution: (a)(i)(10pts) Note that

$$(3x-1)(x+2)+7x(x+1)=(3x^2+6x-x-2)+(7x^2+7x)=\boxed{10x^2+12x-2}$$

(a)(ii)(10pts) Note

$$\begin{aligned}(3x+2)^{1/3}(2)(4x-5)(4)+(4x-5)^2\left(\tfrac{1}{3}\right)(3x+2)^{-2/3}(3) &= (3x+2)^{-2/3}(4x-5)\big[8(3x+2)+(4x-5)\big]\\ &= (3x+2)^{-2/3}(4x-5)\big[24x+16+4x-5\big]\\ &= \boxed{\frac{(4x-5)(28x+11)}{(3x+2)^{2/3}}}\end{aligned}$$

(b)(8pts) Here we have

$$1+3x^2=-5x\Rightarrow 3x^2+5x+1=0\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-5\pm\sqrt{5^2-4(3)(1)}}{2(3)}=\boxed{\frac{-5\pm\sqrt{13}}{6}}$$

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2. (24pts) The following problems are not related. Show all work.

(a)(10pts) Solve the equation $36x^4-13x^2+1=0$. Show all work.

(b)(10pts) Factor the polynomial completely: $x^5-4x^3+8x^2-32$ $\big[$*Hint:* $a^3\pm b^3=(a\pm b)(a^2\mp ab+b^2)\big]$

(c)(4pts) Which of the choices below is the equivalent to $\dfrac{\frac{x}{x+2}-\frac{4}{x+2}}{x-3-\frac{6}{x+2}}$? **Choose only <u>one</u> answer.** *No justification necessary, copy down the entire answer. If you do not copy down the entire answer, points will be deducted.*

(A) $\dfrac{x-4}{x-9}$ (B) $\dfrac{1}{x+3}$ (C) $\dfrac{x-4}{x^2-x}$ (D) $\dfrac{1}{x-4}$ (E) None of these

Solution: (a)(10pts) Note that

$$36x^4-13x^2+1=0\Rightarrow(9x^2-1)(4x^2-1)=0\Rightarrow x^2=\frac{1}{9}\text{ or }x^2=\frac{1}{4}\Rightarrow\boxed{x=\pm\frac{1}{2},\pm\frac{1}{3}}$$

(b)(10pts) Using factoring by grouping and the hint we have

$$\begin{aligned}x^5-4x^3+8x^2-32 &= x^3(x^2-4)+8(x^2-4)\\ &= (x^2-4)(x^3+8)=(x-2)(x+2)\cdot(x+2)(x^2-2x+4)=\boxed{(x-2)(x+2)^2(x^2-2x+4)}\end{aligned}$$

(c)(4pts) $\boxed{\text{Choice B.}}$ Here we have

$$\frac{\dfrac{x}{x+2}-\dfrac{4}{x+2}}{x-3-\dfrac{6}{x+2}}=\frac{\dfrac{x-4}{x+2}}{\dfrac{(x-3)(x+2)-6}{x+2}}=\frac{x-4}{x+2}\cdot\frac{x+2}{(x^2-x-6)-6}=\frac{x-4}{x^2-x-12}=\frac{x-4}{(x-4)(x+3)}=\frac{1}{x+3}\Rightarrow \text{(B)}$$

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3. (20pts) The following problems are not related. Show all work.

(a)(10pts) Find all solutions of the equation $2\sin^2(\theta)=1-\sin(\theta)$ that are in the interval $0\le\theta\le 2\pi$.

(b)(10pts) (i) (2pts) What is the domain of $f(x)=\left|\dfrac{x}{x^2-2}\right|$? Give your answer in interval notation.

(ii) (8pts) Write down the piecewise definition of the function $f(x)=\left|\dfrac{x}{x^2-2}\right|$.

Solution: (a)(10pts) Note that

$$2\sin^2(\theta)=1-\sin(\theta)\Rightarrow 2\sin^2(\theta)+\sin(\theta)-1=0\Rightarrow \big(2\sin(\theta)-1\big)\big(\sin(\theta)+1\big)=0\Rightarrow \sin(\theta)=\frac{1}{2}\text{ or }\sin(\theta)=-1$$

and note that

$$\sin(\theta)=\frac{1}{2}\text{ or }\sin(\theta)=-1\Rightarrow\theta=\frac{\pi}{6},\frac{5\pi}{6}\text{ or }\theta=\frac{3\pi}{2}\Rightarrow\boxed{\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}}$$

(b)(i)(2pts) The domain requires $x^2-2\neq 0\Rightarrow x\neq\pm\sqrt{2}$, that is, the domain is $\boxed{(-\infty,-\sqrt{2})\cup(-\sqrt{2},\sqrt{2})\cup(\sqrt{2},\infty).}$

(b)(ii)(8pts) Note that $\dfrac{x}{x^2-2}=\dfrac{x}{(x-\sqrt{2})(x+\sqrt{2})}$ which is non-negative if $-\sqrt{2}\le x\le 0$ and if $x\ge\sqrt{2}$ and is negative if $x<-\sqrt{2}$ or if $0<x<\sqrt{2}$ thus we have

$$\boxed{f(x)=\left|\frac{x}{x^2-2}\right|=\begin{cases}\dfrac{x}{x^2-2}, & \text{if } -\sqrt{2}<x\le 0\text{ or if }x>\sqrt{2}\\ \dfrac{-x}{x^2-2}, & \text{if } x<-\sqrt{2}\text{ or if }0<x<\sqrt{2}\end{cases}}$$

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4. (28pts) The following problems are not related. Show all work. Simplify your answers

(a)(12pts) Use the formula $\tan(u-v)=\dfrac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$ to find $\tan\left(\dfrac{5\pi}{12}\right)$. (Use $\frac{5\pi}{12}=\frac{3\pi}{4}-\frac{\pi}{3}$.)

(b)(12pts) If $\csc(\phi)=-\dfrac{4}{3}$, find the exact value of $\tan(\phi)$ where $\dfrac{3\pi}{2}<\phi<2\pi$.

(c)(4pts) Which of the choices below is the solution of the equation $\sec(\beta)=2$ where $-\pi\le\beta\le\pi$? **Choose only one answer.** *No justification necessary, copy down the entire answer. If you do not copy down the entire answer, points will be deducted.*

(A) $\dfrac{-\pi}{6},\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{3},\dfrac{5\pi}{3}$ (C) $\dfrac{2\pi}{6},\dfrac{4\pi}{3}$ (D) $\dfrac{-\pi}{3},\dfrac{\pi}{3}$ (E) None of these

Solution: (a)(12pts) Note

$$\tan\left(\frac{5\pi}{12}\right)=\tan\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)=\frac{\tan\left(\frac{3\pi}{4}\right)-\tan\left(\frac{\pi}{3}\right)}{1+\tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}=\boxed{\frac{-1-\sqrt{3}}{1-\sqrt{3}}}=2+\sqrt{3}$$

(b)(12pts) Note that

$$\csc(\phi) = -\frac{4}{3} = \frac{\text{hyp}}{\text{opp}} \Rightarrow \text{adj}^2 = \text{hyp}^2 - \text{opp}^2 = 16 - 9 = 7 \Rightarrow \text{adj} = \pm\sqrt{7}$$

and since $\frac{3\pi}{2} < \phi < 2\pi$, that is, ϕ is in the 4th Quadrant the tangent is negative and so we have $\boxed{\tan(\phi) = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}.}$

(c)(4pts) $\boxed{\text{Choice D.}}$ Here we have

$$\sec(\beta) = 2 \Rightarrow \cos(\beta) = \frac{1}{2} \Rightarrow \beta = -\frac{\pi}{3}, \frac{\pi}{3} \in [-\pi, \pi] \Rightarrow \text{(D)}$$
