INSTRUCTIONS: Outside paper and electronic devices are **not** permitted. Exam is worth 100 points. Neatness counts. Unless indicated, answers with no supporting work may receive no credit. $\boxed{\text{BOX}}$ your final answers.

1. (16 points) Consider the functions: $g(x) = 2 - 2x$ and $h(x) = x^2 - 5x + 4$

   (a) Find the domain of $f(x) = 2 + \dfrac{g(x)}{h(x)}$. Answer in interval notation.

   (b) Factor and reduce $(g/h)(x)$

   (c) Name the vertex of $h(x)$

   Solution: ANS:

   (a) $f(x) = 2 + \frac{2(1-x)}{(x-4)(x-1)} \implies \boxed{(-\infty, 1) \cup (1, 4) \cup (4, \infty)}$

   (b) $\dfrac{-2(x-1)}{(x-1)(x-4)} = \boxed{\frac{-2}{x-4}}$

   (c) $\dfrac{-b}{2a} \implies x = \dfrac{5}{2} \implies y = \left(\dfrac{5}{2}\right)^2 - 5 \cdot \left(\dfrac{5}{2}\right) + 4 = -\dfrac{9}{4}$, $\boxed{\text{vertex is at } (\frac{5}{2}, -\frac{9}{4})}$

2. (12 points) The following questions are not necessarily related:

   (a) Consider the function $f(x) = \sqrt{x+2}$. Find the equation of the line between $f(7)$ and $f(14)$. Answer in slope-intercept form.

   (b) Simplify: $\dfrac{3x^2 - x - 4}{x^2 - 1}$.

   Solution: ANS:

   (a) $f(7) = 3$ and $f(14) = 4$, so between $(7,3)\&(14,4)$ we have m= $\frac{1}{7}$, and $y - 3 = \frac{1}{7}(x - 7) \implies$ $\boxed{y = \frac{1}{7}x + 2}$

   (b) $\dfrac{(3x-4)(x+1)}{(x-1)(x+1)} = \boxed{\dfrac{3x-4}{x-1}}$

3. (16 points) The following are not necessarily related:

   (a) Consider the function $f(x) = \dfrac{1}{\sqrt{x}}$:

   Simplify the following expression and rationalize the numerator: $\dfrac{f(x+h) - f(x)}{h}$.

   (b) Given that $\tan(\theta) = \dfrac{1}{x}$ and $\pi < \theta < \dfrac{3\pi}{2}$, what is $\csc(\theta)$ in terms of x?

   Solution: ANS:

(a) $\dfrac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \dfrac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \dfrac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}} = \dfrac{x - x - h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}\sqrt{x+h})} = \boxed{\dfrac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}}$

(b) $\tan(\theta) = \dfrac{1}{x} \implies r = \sqrt{x^2+1}$, and Quadrant III implies $y < 0$, so $\boxed{\csc(\theta) = \frac{r}{y} = -\sqrt{x^2+1}}$

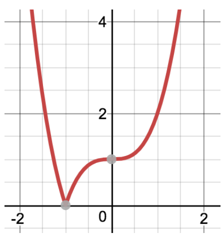
4. (24 points) Sketch the following equations and label any intercepts:

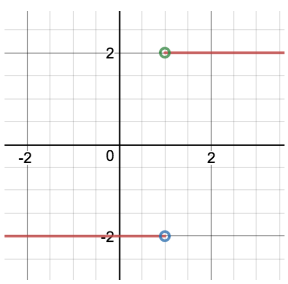
(a) $f(x) = |x^3 + 1|$.

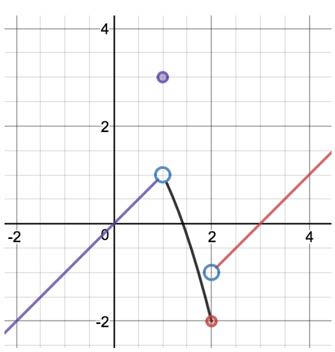
(b) $y = \dfrac{|2x-2|}{x-1}$.

(c) $f(x) = \begin{cases} x & x < 1 \\ 3 & x = 1 \\ 2 - x^2 & 1 < x \leq 2 \\ x - 3 & x > 2 \end{cases}$

Solution: ANS:

(a) 

(b) 

(c) 

5. (20 points) The following questions are not necessarily related:

(a) Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation $\sin(x) = \tan(x)$

(b) If $\sin(x) = \frac{1}{3}$ and $\sec(y) = \frac{5}{4}$, where x and y lie between 0 and $\dfrac{\pi}{2}$, evaluate the expression: $\cos(x - y)$.

Solution: ANS:

(a) $\sin(x) = \tan(x) \implies \sin(x) - \dfrac{\sin(x)}{\cos(x)} = 0 \implies \dfrac{\sin(x)\cos(x) - \sin(x)}{\cos(x)} = 0 \implies \sin(x)[\cos(x) - 1] = 0$
$\sin(x) = 0 \implies x = 0, \pi, 2\pi$ and $\cos(x) = 1 \implies x = 0, 2\pi$. Therefore, $\boxed{x = 0, \pi, 2\pi}$

(b) $\sin(x) = \frac{1}{3} \implies \cos(x) = \frac{2\sqrt{2}}{3}$ and $\cos(y) = \frac{4}{5} \implies \sin(y) = \frac{3}{5}$,
therefore, $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y) = \frac{2\sqrt{2}}{3} \cdot \frac{4}{5} + \frac{1}{3} \cdot \frac{3}{5} = \boxed{\dfrac{8\sqrt{2} + 3}{15}}$

6. (12 points) The following questions are multiple choice and no work is required to be shown. Just write the corresponding letter of the most appropriate answer in your blue book.

(a) Consider the function $f(x) = \sqrt{x + 2}$. The expression $f(x + 1) - 2$ will shift every point on $f(x)$...

i. one unit left and two units up.

ii. one unit right and two units up.

iii. one unit right and two units down.

iv. three units left and two units up.

v. none of the above.

(b) The expression: $\left[\dfrac{\sin(3x)}{3x} \cdot \dfrac{\sin(5x)}{5x} \cdot 15\right]$ can be reduced to what?

i. $\left[\dfrac{\sin(x)\sin(x)}{x^2} \cdot 15\right]$

ii. $\left[\dfrac{\sin^2(15x)}{x^2} \cdot \dfrac{15}{15}\right]$

iii. $\left[\dfrac{\sin(3x)\sin(5x)}{x^2}\right]$

iv. [15]

v. None of the above

(c) Choose a trigonometric expression that is equivalent to: $y = \left[\dfrac{x\cot(2x)}{5}\right]$

i. $y = \dfrac{2x}{5\tan(x)}$

ii. $y = \dfrac{5x}{\tan(2x)}$

iii. $y = \dfrac{5x\cos(x)}{\sin(x)}$

iv. $y = \dfrac{x\cos(2x)}{5\sin(2x)}$

v. None of the above

Solution:

(a) v
(b) iii
(c) iv