INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from another person are not permitted while taking the exam. The final exam is worth 150 points.

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

iii. WRITE YOUR NAME ON THE NEXT PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND FOLLOW PROCTOR INSTRUCTIONS IN UPLOADING YOUR EXAM WITH SUPPORTING WORK TO GRADESCOPE. ONLY WORK THAT'S SUBMITTED TO GRADESCOPE WILL BE GRADED.

Formulas that may be useful:

1. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

10. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. Circle: $(x - h)^2 + (y - k)^2 = r^2$

3. Arc length: $s = r\theta$

11. Area of a sector: $A = \dfrac{1}{2}r^2\theta$

4. $\sin(a - b) = \sin a \cos b - \sin b \cos a$

12. $\sin(a + b) = \sin a \cos b + \sin b \cos a$

5. $\cos(a - b) = \cos a \cos b + \sin a \sin b$

13. $\cos(a + b) = \cos a \cos b - \sin a \sin b$

6. $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$

14. $\sin(2\theta) = 2 \sin \theta \cos \theta$

7. $\cos(2\theta) = 2 \cos^2 \theta - 1$

15. $\cos(2\theta) = 1 - 2 \sin^2 \theta$

8. $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos \theta}{2}}$

16. $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + \cos \theta}{2}}$

9. $\sin^2(\theta) = \dfrac{1 - \cos(2\theta)}{2}$

17. $\cos^2(\theta) = \dfrac{1 + \cos(2\theta)}{2}$

Name: _____

1. Re-arrange the following list of numbers to go from smallest first to largest last. No justification is needed for this problem. (4 pts)

$$e^2, \ \sqrt{2}, \ \frac{1}{\frac{1}{120}}, \ \sqrt{3}, \ |3 - \pi|, \ 2^2, \ |-10|$$

2. Simplify the following: (12 pts)

(a) $\dfrac{1}{3} - \dfrac{\frac{5}{2}}{10} + 2^{-1}$

(b) $\left(9x^3 y^{-5}\right)\left(\dfrac{1}{3}x^{-5}\left(zy^2\right)^3 z^0\right)$

(c) $\left(\dfrac{x^{-1/3}y^{1/2}}{x^{1/3}y^{1/4}}\right)^{-1}$

(d) $\sqrt{12a^2 y}$

3. Simplify the following: (8 pts)

   (a) $\log_4(16) - 10^{2\log(6)} + \ln(1)$

   (b) $\dfrac{1 - \frac{1}{x-1}}{\frac{1}{x-1} + 1}$

4. Solve the following equations for x: (10 pts)
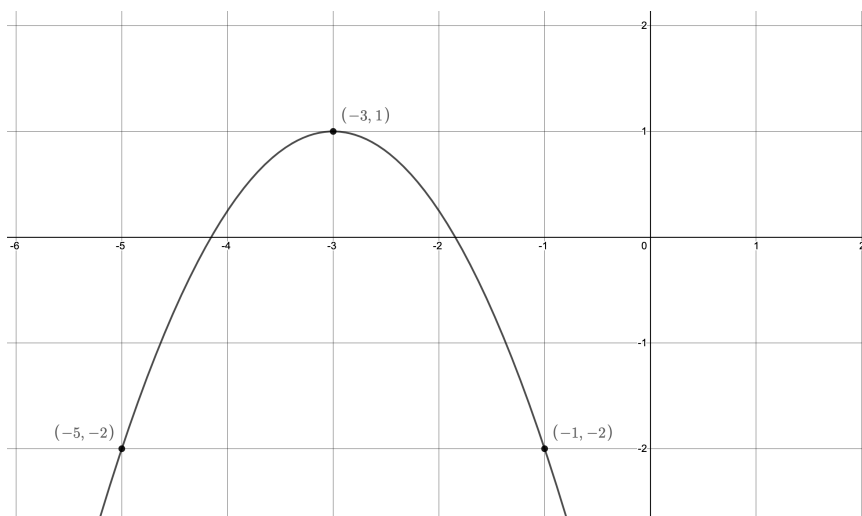
   (a) $(x - 4)^2 = 5$

   (b) $\dfrac{3}{x - 2} + \dfrac{2}{x + 2} = 1$

5. Solve the following equations for x: (10 pts)

   (a) $\log_2(x) = 1 - \log_2(x - 1)$

   (b) $1 = x + \sqrt{3 - x}$

6. The following graph has equation of the form $y = a(x - h)^2 + k$. Find the equation of the function whose graph is given. (5 pts)

7. Consider the function $f(x) = \dfrac{x}{4x^3 - 16x}$. Answer the following: (12 pts)

(a) Find the domain of $f(x)$.

(b) Determine whether $f(x)$ is odd, even, or neither. Make sure to justify your answer for credit.

(c) Find the x-coordinate of any hole(s). If there are none write NONE.

(d) Find the y-coordinate of any hole(s). If there are none write NONE.

(e) Find any horizontal/slant asymptotes. If there are none write NONE.

8. A hot cup of coffee is placed in a $70°$ F dining room. The temperature of the coffee, T in degrees Fahrenheit, as a function of time, t in minutes, is modeled by Newton's law of cooling and is given by the function: $T(t) = 70 + 130e^{-.05t}$. (11 pts)

(a) Find the temperature of the coffee after 100 minutes. Leave your answer in exact form, **do not** attempt to approximate with a decimal value.

(b) How long until the coffee reaches $110°$ F? Leave your answer in exact form, **do not** attempt to approximate with a decimal value.
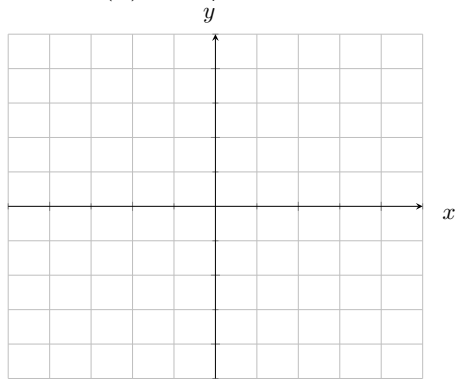
(c) According to the model, what is the initial temperature of the coffee?

9. Sketch the graph of each of the following. Make sure to label relevant value(s) on your axe(s). (12 pts)
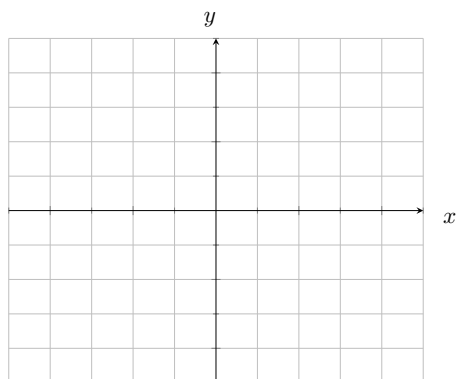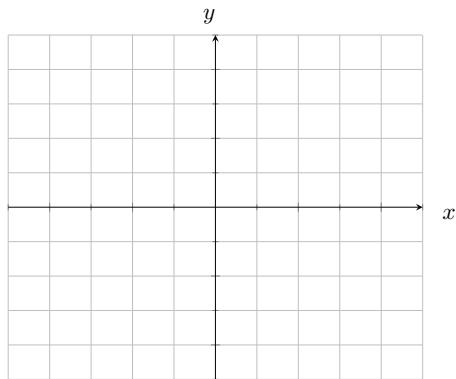
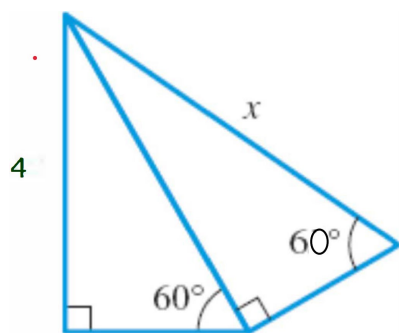(a) $f(x) = 2^{x-1}$
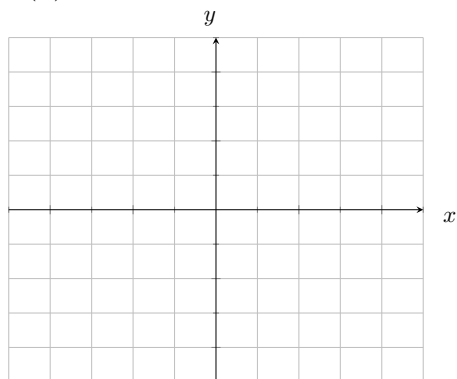
(b) $g(x) = -\sqrt{x}$

(c) $h(x) = \sin x$ on the restricted domain $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

(d) $i(x) = \sin^{-1} x$

10. Solve for x as depicted in the image (6 pts)

11. Find the exact value: (15 pts)

(a) $\cos\left(\dfrac{7\pi}{6}\right)$

(d) $\arccos\left(\cos\left(\dfrac{\pi}{4}\right)\right)$
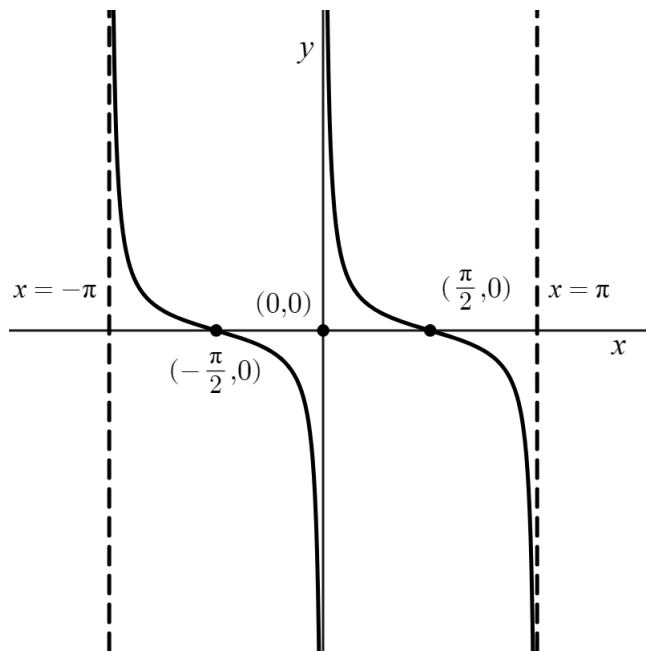
(b) $\csc\left(\dfrac{4\pi}{3}\right)$

(e) $\sec\left(\sin^{-1}\left(\dfrac{1}{2}\right)\right)$

(c) $\tan^{-1}(-1)$

12. Verify the identity: $\cos^2\theta\cot\theta = \cot\theta - \cos\theta\sin\theta$ (5 pts)

(Hint: Start with the right hand side and recall the definition of $\cot\theta$)

13. Use the graph of the function $f(x)$ below, with $f(0) = 0$ and with domain $(-\pi, \pi)$, to answer the following questions. No justification of your answers is needed for this problem. (10 pts)

(a) Solve the equation $f(x) = 0$.

(b) Solve the inequality $f(x) < 0$ and give your answer in interval notation.

(c) Identify the restriction of domain of $f(x)$ so that the range is preserved and the graph is one-to-one.

(d) Is $f(x)$ odd, even, or neither?

14. Find all solutions to the following equations: (10 pts)

(a) $\cos(4\theta) = \dfrac{1}{2}$

(b) $\sin\theta \tan\theta - 2\tan\theta = 0$

15. Find the exact value for each: (8 pts)

(a) $\cos\left(\dfrac{\pi}{6}\right)\cos\left(\dfrac{\pi}{12}\right) - \sin\left(\dfrac{\pi}{6}\right)\sin\left(\dfrac{\pi}{12}\right)$

(b) $\sin\left(15^o\right)$

16. For $f(x) = 3\cos\left(x - \dfrac{\pi}{3}\right)$ (12 pts)

    (a) Identify the amplitude.

    (b) Identify the period.

    (c) Identify the phase shift.

    (d) Sketch two cycles of $f(x)$ labeling relevant y-values and at least two values on the x-axis.

End of Final Exam. Have an awesome summer!