INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from another person are not permitted while taking the exam. The exam is worth 100 points.

Potentially useful formulas:

(i) Equation of a circle: $(x-h)^2+(y-k)^2=r^2$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

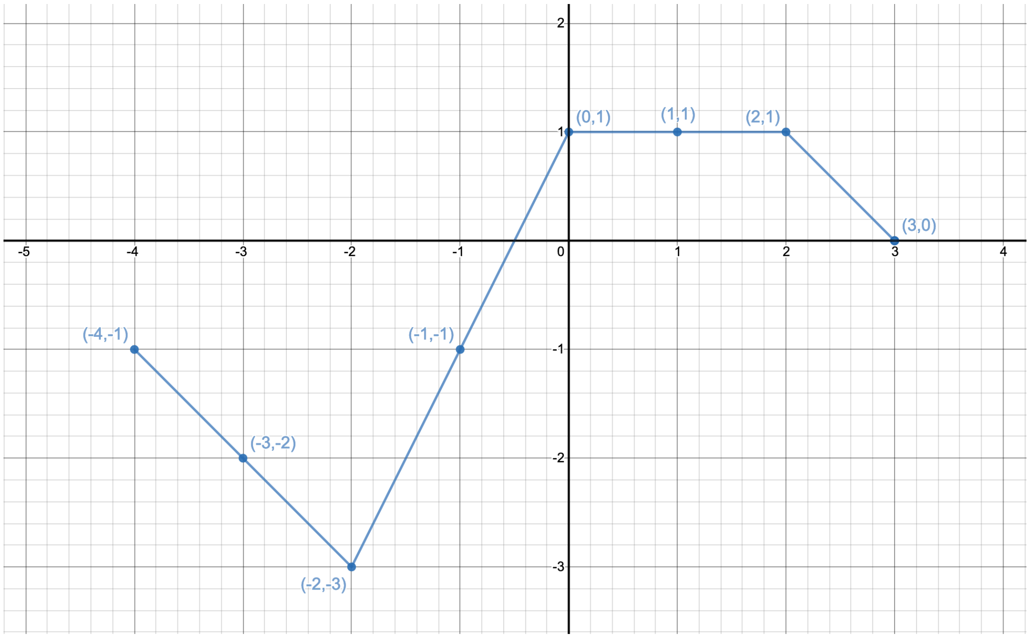
ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

iii. WRITE YOUR NAME ON THE NEXT PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND FOLLOW PROCTOR INSTRUCTIONS IN UPLOADING YOUR EXAM WITH SUPPORTING WORK TO GRADESCOPE. ONLY WORK THAT'S SUBMITTED TO GRADESCOPE WILL BE GRADED.

Name: ____

1. Refer to the given graph of $f(x)$ to answer the following: (9 pts)



(a) Find the domain of $f(x)$. Express your answer in interval notation

Solution:

$\boxed{[-\mathbf{4}, \mathbf{3}]}$

(b) Find the range of $f(x)$. Express your answer in interval notation

Solution:

$\boxed{[-\mathbf{3}, \mathbf{1}]}$

(c) Find $(f+f)(-2)$

Solution:

$(f+f)(-2) = f(-2) + f(-2) = \boxed{-\mathbf{6}}$

(d) Find $(f \circ f)(3)$

Solution:

$f\left(f(3)\right) = f(0) = \boxed{\mathbf{1}}$

(e) Find $|f(-4)|$

Solution:

$|-1| = \boxed{\mathbf{1}}$

(f) Solve $f(x) = -3$

Solution:

$\boxed{\mathbf{-2}}$

(g) Find x-values for which $f(x) \leq -1$

Solution:

$\boxed{[\mathbf{-4, -1}]}$

(h) Find the net change of $f(x)$ from $x = 0$ to $x = 3$

Solution:

$f(3) - f(0) = 0 - 1 = \boxed{\mathbf{-1}}$

(i) Is $f(x)$ one-to-one? Briefly justify your answer

Solution:

$\boxed{\textbf{NO}}$ because $f(x)$ doesn't pass the Horizontal line test.

2. Find the domain of the following functions: (15 pts)

(a) $f(x) = \sqrt{x^2 - x - 12}$

Solution:

Since the square root of a negative quantity does not exist in the real numbers,

$$x^2 - x - 12 \geq 0 \tag{1}$$
$$(x-4)(x+3) \geq 0 \tag{2}$$

Setting the left side equal to zero we get two values that make the left side zero: $x = -3$ and $x = 4$. Placing these on a number line and picking test values we obtain the following chart

From this, we find the domain to be $\boxed{(-\infty, \mathbf{-3}] \cup [\mathbf{4}, \infty)}$

(b) $g(x) = x^2 + \sqrt[3]{-x}$

Solution:

The 1st term is a polynomial and the 2nd is a cube root function, both of which are valid for all real numbers. Hence the domain is $\boxed{(-\infty, \infty)}$

(c) $h(x) = \dfrac{x-1}{x\sqrt{9-x}}$

Solution:

Since the square root of a negative quantity does not exist in the real numbers, and the denominator of a fraction can't be 0, we require that both $9 - x > 0$ **and** $x \neq 0$. Hence the domain is $\boxed{(-\infty, \mathbf{0}) \cup (\mathbf{0}, \mathbf{9})}$

3. Consider $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$ and answer the following: (6 pts)

   (a) Find $(f \circ g)(x)$.

   Solution:

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) && (3) \\ &= f\left(x^2 + 1\right) && (4) \\ &= \boxed{\sqrt{x^2 + 1}} && (5) \end{aligned}$$

   (b) Find the domain of $(f \circ g)(x)$.

   Solution:

   Since $x^2 + 1$ is never negative, all real numbers are allowed in the operations we performed. Thus, the domain is $\boxed{(-\infty, \infty)}$

4. Find the equation of a parabola which has vertex at $(-2, 5)$ and passes through the point $(0, 7)$: (4 pts)

   Solution:

   The equation of a parabola with vertex (h, k) in standard form is $y = a(x - h)^2 + k$
   Hence we have

   $y = a\left(x - (-2)\right)^2 + 5 = a\left(x + 2\right)^2 + 5$

   Since it passes through $(0, 7)$ we can write

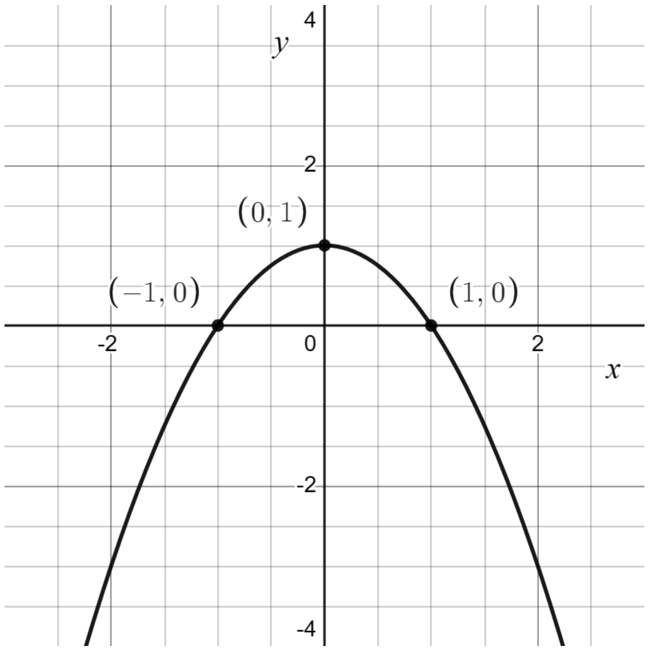
$$\begin{aligned} 7 &= a\left(0 - (-2)\right)^2 + 5 && (6) \\ \therefore a &= \frac{1}{2} && (7) \end{aligned}$$

   Hence the equation of the parabola is

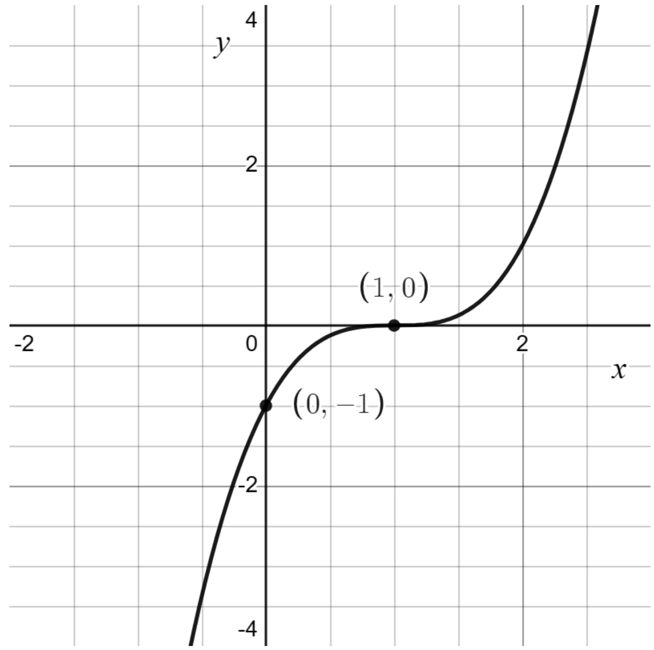
$$\boxed{y = \frac{1}{2}\left(x + 2\right)^2 + 5}$$

5. Sketch the graph of the following (graph each function on a **separate** set of axes). Label values on your axes: (19 pts)

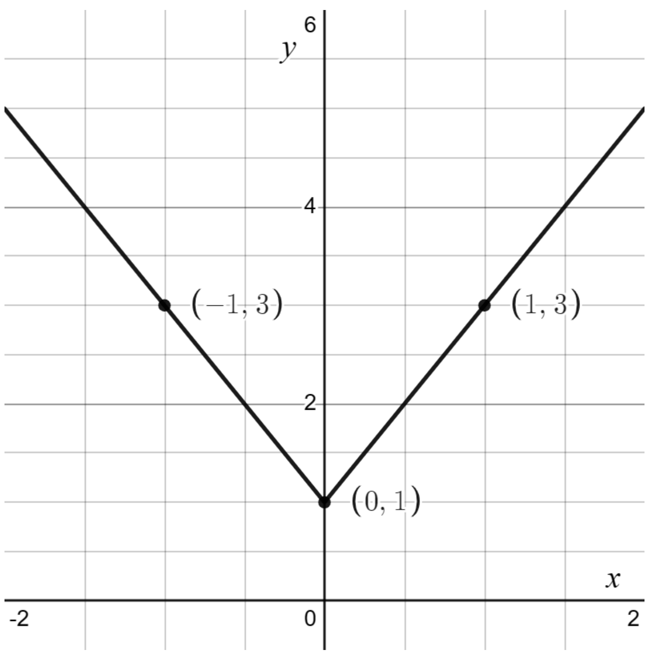
(a) $f(x) = -x^2 + 1$



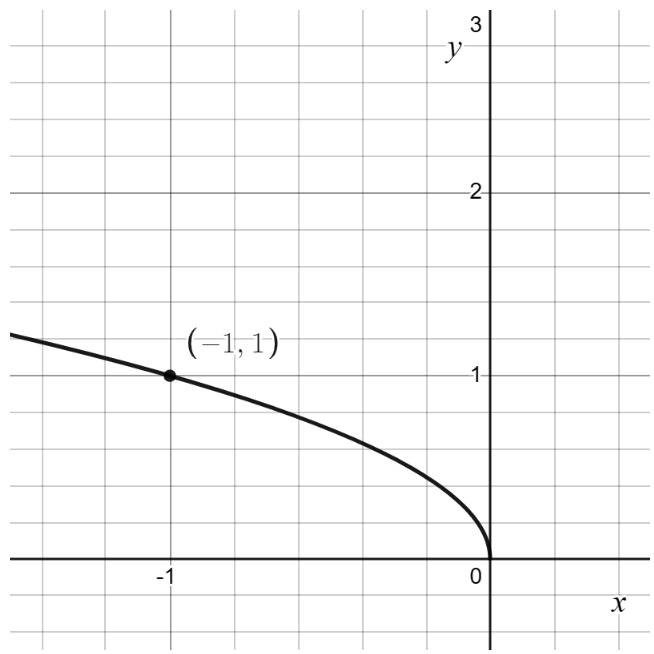
(b) $g(x) = (x - 1)^3$



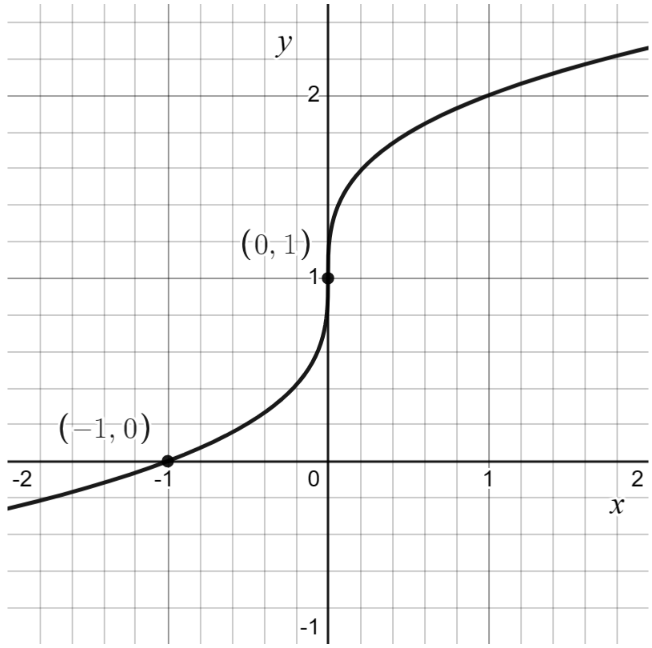
(c) $h(x) = 2|x| + 1$



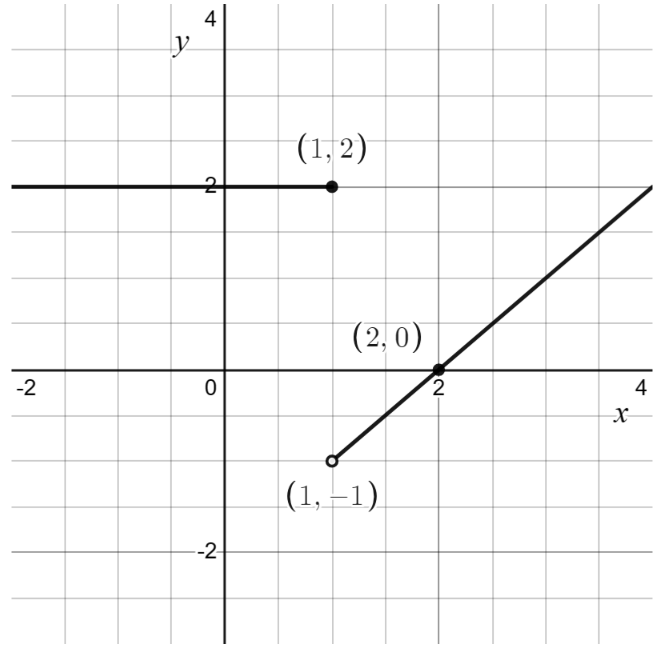
(d) $i(x) = \sqrt{-x}$



(e) $j(x) = \sqrt[3]{x} + 1$



(f) $k(x) = \begin{cases} 2 & x \leq 1 \\ x - 2 & x > 1 \end{cases}$



6. Write the equation of a straight line that is perpendicular to the line $y = \dfrac{1}{3}x + 96$ and passes thru the point$(1, -4)$. (4 points)
Solution:
Since our line is perpendicular to the given straight line, the slope of our line is
$m = -\dfrac{1}{\left(\frac{1}{3}\right)} = -3$
Hence the equation of the line in point slope form is given by

$$\boxed{(\mathbf{y} + \mathbf{4}) = -\mathbf{3}(\mathbf{x} - \mathbf{1})}$$

7. The following are unrelated: (6 pts)

(a) Find the equation of the line that is parallel to the x-axis and passes through the point $(-2, 5)$.

Solution: Since the line is parallel to x axis, the y coordinate never changes. Hence the equation is $\boxed{\mathbf{y} = \mathbf{5}}$

(b) $f(x)$ is an even function with domain $(-\infty, \infty)$. The point $(3, 5)$ lies on its graph. Which of the following points must also lie on its graph?
(i) $(-3, 5)$ (ii) $(-3, -5)$ (iii) $(3, -5)$
Solution:
$\boxed{(-\mathbf{3}, \mathbf{5})}$

(c) $g(x)$ is symmetric about the origin, with domain $(-\infty, \infty)$. The point $(3, 5)$ lies on its graph. Which of the following points must also lie on its graph?
(i) $(-3, 5)$ (ii) $(-3, -5)$ (iii) $(3, -5)$
Solution:
$\boxed{(-\mathbf{3}, -\mathbf{5})}$

8. A bored pre-calculus student stands on a 3 ft high stool, and throws a stone vertically upwards with an initial speed of 32 ft/s. The height of that stone above the ground, in ft, is given as a function of time, in seconds, by $y(t) = 32t - 16t^2 + 7$. Answer the following: (6 pts)

   (a) At what time is the stone at its maximum height?

   Solution:

   The parabola $y = -16t^2 + 32t + 7$ opens down, and thus have a maximum at the vertex

$$\begin{aligned} h &= -\frac{32}{2(-16)} && (8) \\ &= 1 && (9) \end{aligned}$$

   Hence the maximum height is achieved at $\boxed{1s}$

   (b) What is the maximum height of the stone, measured from the ground?

   Solution:

   The maximum height is the y-coordinate of the vertex is given by $k = y(h) = \boxed{23ft}$

9. The points $(-1, 2)$ and $(1, 4)$ lie directly opposite each other on a circle such that the distance between the two points gives the diameter of the circle. Answer the following: (9 pts)

   (a) What is the radius of the circle?

   Solution:

   The diameter of the circle is the distance between the diametrically opposite points, which we obtain by the distance formula:

$$\begin{aligned} d &= \sqrt{(1-(-1))^2 + (4-2)^2} && (10) \\ &= \sqrt{8} && (11) \\ &= 2\sqrt{2} && (12) \end{aligned}$$

   Hence the radius of the circle is given by $r = \frac{d}{2} = \boxed{\sqrt{2}}$

   (b) Where is the center of the circle?

   Solution:
   The Center of the circle is the midpoint of the given points, given by the midpoint formula:

$$\begin{aligned} (h, k) &= \left(\frac{-1+1}{2}, \frac{2+4}{2}\right) && (13) \\ &= \boxed{(0, 3)} && (14) \end{aligned}$$

   (c) Using the above, write down the equation of the circle in standard form

Solution:
The equation of the circle in standard form is

$$(x-0)^2 + (y-3)^2 = \left(\sqrt{2}\right)^2 \tag{15}$$
$$x^2 + (y-3)^2 = 2 \tag{16}$$

$$\boxed{\mathbf{x^2 + (y-3)^2 = 2}}$$

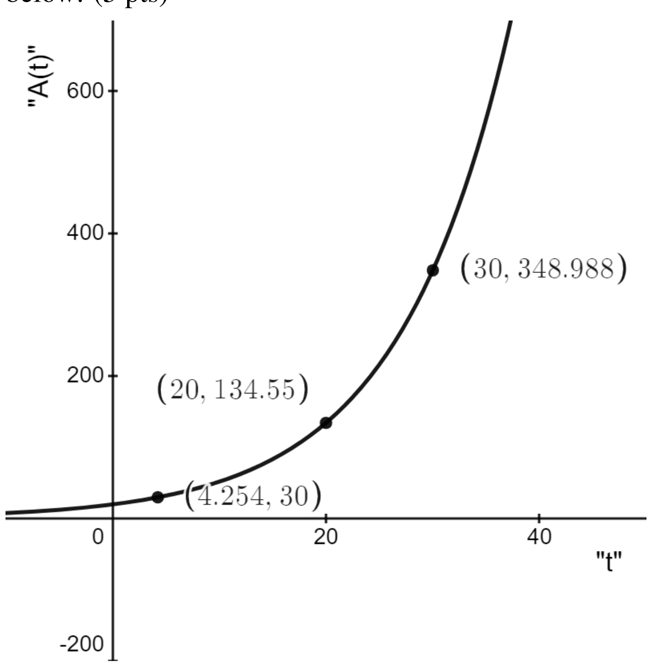
10. Find the inverse of the function $f(x) = \dfrac{1}{2x^3} + 7$ (4 pts)

Solution:
In order to find the inverse, we first call $f(x) = y = \frac{1}{2x^3} + 7$. Then we interchange x and y and solve for y in terms of x

$$x = \frac{1}{2y^3} + 7 \tag{17}$$
$$\frac{1}{2y^3} = x - 7 \tag{18}$$
$$y^3 = \frac{1}{2(x-7)} \tag{19}$$
$$y = \sqrt[3]{\frac{1}{2(x-7)}} \tag{20}$$

Hence the inverse is $$f^{-1}(x) = \boxed{\sqrt[3]{\frac{1}{2(x-7)}}}$$

11. The small amount you put in your savings account a few years ago is growing rapidly(Yay!). Below is a graph of your balance $A(t)$ in dollars, as a function of time t in years. Use the graph to answer the parts below: (5 pts)



(a) Find $A^{-1}(30)$.

Solution:

Since $A(4.254) = 30$ we have $T^{-1}(30) = \boxed{4.254 \ \ years}$

(b) Use a short sentence to interpret the meaning of your answer to part (a).

Solution:

$\boxed{\text{After 4.254 years the amount will be 30 dollars}}$

12. For $f(x) = -x + 5$ find the following: (5 pts)

(a) $f(a)$

Solution:

$\boxed{\mathbf{-a+5}}$

(b) $f(a+h)$

Solution:

$\boxed{\mathbf{-(a+h)+5}}$

(c) $\dfrac{f(a+h)-f(a)}{h}$

Solution:

$$\frac{f(a+h)-f(a)}{h} = \frac{-(a+h)+5-(-a+5)}{h} \tag{21}$$

$$= \frac{-a-h+5+a-5}{h} \tag{22}$$

$$= \frac{-h}{h} \tag{23}$$

$$= \boxed{-1} \tag{24}$$

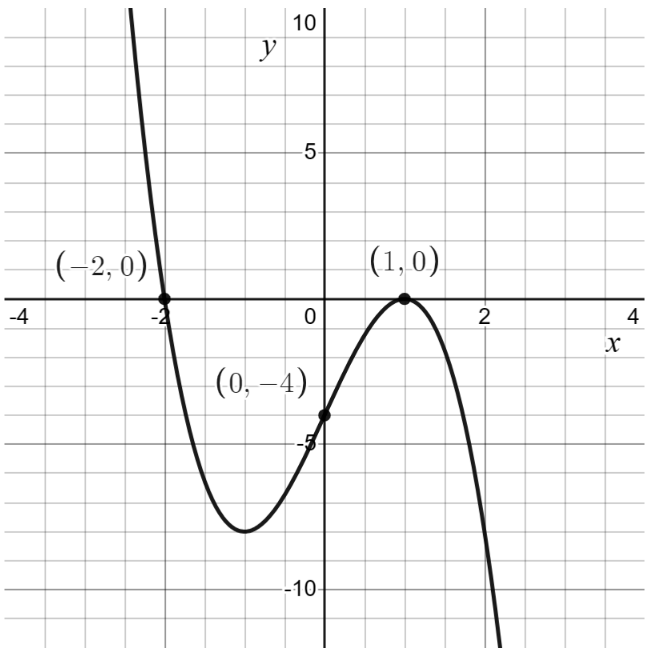
13. The graph of a polynomial, $P(x)$, has the following properties: (8 pts)

i. The graph has end behavior that is consistent with the end behavior of $y = -x^3$ (In arrow notation: $P(x) \to -\infty$ as $x \to \infty$ and $P(x) \to \infty$ as $x \to -\infty$).
ii. The graph crosses at x-intercept $(-2, 0)$.
iii. The graph bounces (touches but does not cross) at x-intercept $(1, 0)$.
iv. The graph has no other x-intercepts.
v. The graph has y-intercept $(0, -4)$.

Answer the following for $P(x)$:

(a) Sketch a graph of $P(x)$ that satisfies **all** of the given information.

Solution:



(b) Find an equation of a polynomial $P(x)$ that satisfies **all** of the given information. You may leave your answer in factored form.

Solution:
From the above information, we see that the $x = -2$ has odd multiplicity (crosses), and $x = 1$ has even multiplicity (bounces). Since the graph has no other x-intercept, the simplest polynomial that would give us that graph would be of the form

$$P(x) = A\,(x - (-2))\,(x - 1)^2 = A(x + 2)\,(x - 1)^2$$

Since it passes through $(0, -4)$ we have

$$-4 = A(0 + 2)\,(0 - 1)^2 \quad (25)$$
$$-4 = 2A \quad (26)$$
$$A = -2 \quad (27)$$

Hence an equation of the polynomial would be $\boxed{\mathbf{P(x) = -2(x + 2)\,(x - 1)^2}}$ which is consistent with the end behavior of $y = -x^3$