1. The following are unrelated: (15 pts)

   (a) Rewrite each of the following without the absolute value symbol:

   i. $|2\pi - 6|$

   Solution:

   Since $\pi > 3$, $2\pi > 6$, $2\pi - 6 > 0$ so $|2\pi - 6| = \boxed{2\pi - 6}$

   ii. $|\sqrt{2} - 2|$

   Solution:

   Since $\sqrt{2} < 2$, $\sqrt{2} - 2 < 0$ hence $|\sqrt{2} - 2| = \boxed{-\left(\sqrt{2} - 2\right)}$ or $\boxed{2 - \sqrt{2}}$

   (b) Use the definition of the distance between two numbers, including absolute value symbol, to write down the distance between $\mathbf{-7}$ and $\mathbf{-4}$, then find the distance.

   Solution:

$$
\begin{aligned}
d(-7,-4) &= |-7-(-4)| &(1)\\
&= |-3| &(2)\\
&= \boxed{3} &(3)
\end{aligned}
$$

   (c) Let x, y, and z be real numbers such that $x < 2$, $y < 0$, and $2 \leq z \leq 4$. Answer the following:

   i. Is $-z^4y^3$ positive, negative, or cannot be determined? No work is needed to justify your answer.

   Solution:

   $\boxed{\text{Positive}}$

   ii. Is x^7y^{23} positive, negative, or cannot be determined? No work is needed to justify your answer.

   Solution:

   $\boxed{\text{Cannot be determined}}$

   iii. Is $z - 5 + y$ positive, negative, or cannot be determined? No work is needed to justify your answer.

   Solution:

   $\boxed{\text{Negative}}$

(d) Add and simplify: $\frac{2}{\frac{9}{7}} + \frac{5}{12} + 7^0$

Solution:

$$\frac{2}{\frac{9}{7}} + \frac{5}{12} + 7^0 = \frac{14}{9} + \frac{5}{12} + 1 \tag{4}$$
$$= \frac{56}{36} + \frac{15}{36} + \frac{36}{36} \tag{5}$$
$$= \boxed{\frac{107}{36}} \tag{6}$$

(e) Simplify: $\frac{|-7-3| + |2|}{2|-4|}$

Solution:

$$\frac{|-7-3| + |2|}{2|-4|} = \frac{10+2}{8} \tag{7}$$
$$= \frac{12}{8} \tag{8}$$
$$= \boxed{\frac{3}{2}} \tag{9}$$

2. The following are unrelated. Leave your answers without negative exponents. (20 pts)

(a) $(-5b^3)^2 7a^{-3}a^6$

Solution:

$$(-5b^3)^2 7a^{-3}a^6 = (-5)^2(b^3)^2 7a^3 \tag{10}$$
$$= 25b^6 7a^3 \tag{11}$$
$$= \boxed{175b^6a^3} \tag{12}$$

(b) Simplify: $\frac{\sqrt{32x^2}}{\sqrt{2\sqrt{16}}}$

Solution:

$$\frac{\sqrt{32x^2}}{\sqrt{2\sqrt{16}}} = \frac{|x|\sqrt{2 \cdot 16}}{\sqrt{2 \cdot 4}} \tag{13}$$
$$= \frac{|x|4\sqrt{2}}{2\sqrt{2}} \tag{14}$$
$$= \boxed{2|x|} \tag{15}$$

(c) Simplify: $\frac{2(x^{-2}y^3)^3}{8x^{-3}y^{-1/3}}$

Solution:

$$\frac{2(x^{-2}y^3)^3}{8x^{-3}y^{-1/3}} = \frac{2x^{-6}y^9}{8x^{-3}y^{-1/3}} \tag{16}$$
$$= \frac{x^{-3}y^{\frac{27}{3}}}{4y^{\frac{1}{3}}} \tag{17}$$
$$= \boxed{\frac{y^{\frac{28}{3}}}{4x^3}} \tag{18}$$

(d) Multiply to rewrite as a polynomial: $\left(\sqrt{x-1}+3\right)\left(\sqrt{x-1}-3\right)$

Solution:

$$\left(\sqrt{x-1}+3\right)\left(\sqrt{x-1}-3\right) = \left(\sqrt{x-1}\right)^2 - 3^2 \tag{19}$$
$$= x-1-9 \tag{20}$$
$$= \boxed{x-10} \tag{21}$$

3. The following are unrelated: (12 pts)

(a) Find the domain of the expression (give your answer in interval notation): $\dfrac{x^2-9}{\sqrt{x}(x-3)}$

Solution: $x-3=0$ results in division by zero so $x=3$ must be excluded from the domain. As well $x>0$ since the square root of a negative number does not exist. So the answer is: $\boxed{(0,3)\cup(\infty)}$

(b) Combine into a single fraction: $\dfrac{1}{x^2-x-2} - \dfrac{3}{x+1}$

Solution:

$$\frac{1}{x^2-x-2} - \frac{3}{x+1} = \frac{1}{(x-2)(x+1)} - \frac{3}{x+1} \tag{22}$$
$$= \frac{1-3(x-2)}{(x-2)(x+1)} \tag{23}$$
$$= \boxed{\frac{7-3x}{(x-2)(x+1)}} \tag{24}$$

(c) Evaluate the expression: $-\dfrac{1}{2}x^2 - x^{-1}$ when $x=-4$

Solution: $-\dfrac{1}{2}(-4)^2 - \dfrac{1}{-4} = -8 + \dfrac{1}{4} = -\dfrac{32}{4} + \dfrac{1}{4} = \boxed{-\dfrac{31}{4}}$

4. The following are unrelated: (12 pts)

(a) Simplify: $\dfrac{3x^2+12}{x^2-5x} \cdot \dfrac{x^2-3x-10}{x^2+4}$

Solution:

$$\frac{3x^2+12}{x^2-5x} \cdot \frac{x^2-3x-10}{x^2+4} = \frac{3(x^2+4)}{x(x-5)} \cdot \frac{(x-5)(x+2)}{x^2+4} \tag{25}$$
$$= \boxed{\frac{3(x+2)}{x}} \tag{26}$$

(b) Simplify the compound fraction: $\dfrac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1}$

Solution:

$$\frac{\frac{3}{x^2} - \frac{1}{x}}{\frac{9}{x^2} - 1} = \frac{\left(\frac{3-x}{x^2}\right)}{\left(\frac{9-x^2}{x^2}\right)} \tag{27}$$

$$= \frac{3-x}{9-x^2} \tag{28}$$

$$= \frac{3-x}{(3-x)(3+x)} \tag{29}$$

$$= \boxed{\frac{1}{3+x}} \tag{30}$$

(c) Factor by grouping: $9x^3 - 18x^2 - 4x + 8$

Solution:

$$9x^3 - 18x^2 - 4x + 8 = 9x^2(x-2) - 4(x-2) \tag{31}$$

$$= (x-2)(9x^2 - 4) \tag{32}$$

$$= (x-2)\left((3x)^2 - 2^2\right) \tag{33}$$

$$= \boxed{(x-2)(3x+2)(3x-2)} \tag{34}$$

5. Is $x = -2$ a solution of the inequality $x^3 - 2x < 2x$? (3 pts)

Solution:

$\boxed{\textbf{No}}$ as $(-2)^3 - 2(-2) = -4 = 2(-2)$

6. Solve each of the following equations. If there are no solutions write NO SOLUTIONS: (15 pts)

(a) $-2x - 4 = 1 + 4x$

Solution:

$6x = -5$ or $\boxed{x = -\frac{5}{6}}$

(b) $\dfrac{3}{x-2} = \dfrac{x}{x^2-4}$

Solution:

Factoring and Multiplying By the LCD, we obtain

$$(x-2)(x+2) \cdot \frac{3}{x-2} = \frac{x}{(x-2)(x+2)} \cdot (x-2)(x+2) \tag{35}$$

$$3(x+2) = x \tag{36}$$

$$3x + 6 = x \tag{37}$$

$$2x = -6 \tag{38}$$

$$\boxed{x = -3} \tag{39}$$

(c) $|2x - 3| = 3$

Solution:

$2x - 3 = 3 \implies \boxed{x = 3}$

$2x - 3 = -3 \implies \boxed{x = 0}$

7. Solve each of the following equations. If there are no solutions write NO SOLUTIONS: (10 pts)

(a) $\sqrt{8-y}+2=y-4$

Solution:

$$\sqrt{8-y}+2=y-4 \quad (40)$$
$$\sqrt{8-y}=y-6 \quad (41)$$
$$8-y=y^2-12y+36 \quad (42)$$
$$y^2-11y+28=0 \quad (43)$$
$$(y-7)(y-4)=0 \quad (44)$$
$$y=4,7 \quad (45)$$

Plugging into the original equation, we find $y=4$ to be extraneous. Hence $\boxed{y=7}$

(b) Solve for h: $P=A+hdg$

Solution:

$$P=A+hdg \quad (46)$$
$$hdg=P-A \quad (47)$$
$$h=\boxed{\frac{P-A}{dg}} \quad (48)$$

8. Solve the following inequalities. Justify your answers by using a number line or sign chart if needed. Answers without full justification will not receive full credit. Express all answers in interval notation. (8 pts)

(a) $-3x+1<6$

Solution:

$$-3x+1<6 \quad (49)$$
$$-3x<5 \quad (50)$$
$$x>-\frac{5}{3} \quad (51)$$

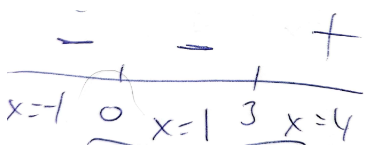
Hence the interval of solution is $\boxed{\left(-\frac{5}{3},\infty\right)}$

(b) $x^3 - 3x^2 \geq 0$

Solution:
We start by factoring the left hand side, and then make use of a number line/sign chart to choose the relevant interval of solution

$$x^3 - 3x^2 \geq 0 \tag{52}$$
$$x^2(x-3) \geq 0 \tag{53}$$

Setting the left side equal to zero we get two values that make the left side zero: $x = 0$ and $x = 3$. Placing these on a number line and picking test values we obtain the following chart



Notice that $x = 0$ is a solution. Hence the solution is $\boxed{0 \cup [3, \infty)}$.

9. Find all the solutions to the following equation, including the complex solutions (Hint: factoring will be important) $z^3 = 1$. (5 pts)

Solution:

$$z^3 - 1 = 0 \tag{54}$$
$$(z-1)(z^2 + z + 1) = 0 \tag{55}$$

From which we conclude that $\boxed{z = 1}$ or

$$z = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} \tag{56}$$
$$= \frac{-1 \pm \sqrt{-3}}{2} \tag{57}$$
$$= \boxed{\frac{-1 \pm \sqrt{3}i}{2}} \tag{58}$$