1. The following are unrelated. (7 pts)

   (a) Place the correct symbol, $<$, $>$, or, $=$ in the space between each of the following pair of numbers.

   i. $\frac{2}{7} \quad \frac{8}{28}$

   Solution:

   Since $\frac{8}{28} = \frac{4 \cdot 2}{4 \cdot 7} = \frac{2}{7}$ then $\boxed{\frac{2}{7} = \frac{8}{28}}$

   ii. $\sqrt{3} \quad \sqrt{2}$

   Solution:

   Since $\sqrt{b} > \sqrt{a}$ when $b > a$ then $\boxed{\sqrt{3} > \sqrt{2}}$.

   (b) Let a and b be real numbers such that $a < 0$ and $b < 0$. Determine whether the following expression is positive, negative, or zero: $-4a^4b^3$.

   Solution:

   Since a^4 is positive and b^3 is negative then $-4a^4b^3$ is a negative number times a positive number times a negative number which is $\boxed{\text{positive}}$.

   (c) **True or False**: $\sqrt{x^2+4} = x+2$. If false, pick a value for x and show, for this value, that the left side does not equal the right side.

   Solution:

   The answer is $\boxed{\text{False}}$ because, for $x = 1$, the left side becomes $\sqrt{1^2+4} = \sqrt{5}$ but on the right side $1+2=3$ and $\sqrt{5} \neq 3$.

2. The following are unrelated. Leave answers without negative exponents. (8 pts)

   (a) Evaluate and simplify: $-\frac{1}{15} + \frac{3}{10} - \frac{1}{\frac{1}{3}} - 4^{-1/2} + \sqrt{9}$

   Solution:

$$
\begin{aligned}
-\frac{1}{15} + \frac{3}{10} - \frac{1}{\frac{1}{3}} - 4^{-1/2} + \sqrt{9} &= -\frac{2}{30} + \frac{9}{30} - 3 - \frac{1}{4^{1/2}} + 3 && (1) \\
&= \frac{7}{30} - \frac{1}{2} && (2) \\
&= \frac{7}{30} - \frac{15}{30} && (3) \\
&= -\frac{8}{30} && (4) \\
&= \boxed{-\frac{4}{15}} && (5)
\end{aligned}
$$

(b) Simplify: $-\dfrac{3x^{-2}}{x^{-3}} + 2(1-x)(x+3)$

Solution:

$$\begin{aligned}
-\frac{3x^{-2}}{x^{-3}} + 2(1-x)(x+3) &= -\frac{3x^3}{x^2} + 2\left(x + 3 - x^2 - 3x\right) && (6)\\
&= -3x + 2x + 6 - 2x^2 - 6x && (7)\\
&= \boxed{-2x^2 - 7x + 6} && (8)
\end{aligned}$$

3. The following are unrelated. (16 pts)

(a) Subtract and simplify: $\dfrac{x+1}{x^2+4x+4} - \dfrac{4}{x^2-2x}$

Solution:

$$\begin{aligned}
\frac{x+1}{x^2+4x+4} - \frac{4}{x^2-2x} &= \frac{x+1}{(x+2)(x+2)} - \frac{4}{x(x-2)} && (9)\\
&= \frac{x(x+1)(x-2)}{x(x+2)(x+2)(x-2)} - \frac{4(x+2)(x+2)}{x(x-2)(x+2)(x+2)} && (10)\\
&= \frac{x(x+1)(x-2) - 4(x+2)(x+2)}{x(x-2)(x+2)^2} && (11)\\
&= \frac{x^3 - x^2 - 2x - 4 - 4x^2 - 16x - 16}{x(x-2)(x+2)^2} && (12)\\
&= \boxed{\frac{x^3 - 5x^2 - 18x - 16}{x(x-2)(x+2)^2}} && (13)
\end{aligned}$$

(b) Simplify: $\dfrac{\sqrt{r^8}}{\left(16r^2\right)^{1/4}}$

Solution:

$$\begin{aligned}
\frac{\sqrt{r^8}}{\left(16r^2\right)^{1/4}} &= \frac{r^4}{16^{1/4}\left(r^2\right)^{1/4}} && (14)\\
&= \frac{r^4}{2r^{1/2}} && (15)\\
&= \boxed{\frac{r^{7/2}}{2}} && (16)
\end{aligned}$$

(c) Simplify: $\dfrac{\frac{2}{x+1}-1}{\frac{2}{x^2}-\frac{2}{x}}$

Solution:

$$\begin{aligned}
\frac{\frac{2}{x+1}-1}{\frac{2}{x^2}-\frac{2}{x}} &= \frac{\frac{2}{x+1}-\frac{x+1}{x+1}}{\frac{2}{x^2}-\frac{2x}{x^2}} && (17)\\
&= \frac{\frac{-x+1}{x+1}}{\frac{2-2x}{x^2}} && (18)\\
&= \frac{-x+1}{x+1}\cdot\frac{x^2}{2-2x} && (19)\\
&= \frac{(1-x)x^2}{(x+1)(2)(1-x)} && (20)\\
&= \boxed{\frac{x^2}{2(x+1)}} && (21)
\end{aligned}$$

(d) Simplify the following: $\ln(1)-\ln(e)+\log(100)-\log_3\left(\frac{1}{9}\right)$ (Your answer should have no logarithms)

Solution:

Solving for one logarithm at a time:

$$\begin{aligned}
&\ln(1) = 0 && (22)\\
&\ln(e) = 1 && (23)\\
&\log(100) = 2 && (24)\\
&\log_3\left(\frac{1}{9}\right) = \log_3\left(3^{-2}\right) = -2 && (25)
\end{aligned}$$

Putting that all together:

$$\begin{aligned}
\ln(1)-\ln(e)+\log(100)+\log_3\left(\frac{1}{9}\right) &= 0-1+2-(-2) && (26)\\
&= \boxed{3} && (27)
\end{aligned}$$

4. The following are unrelated: (6 pts)

(a) Simplify and write in $a+bi$ form: $(2-i)+4+2i-i^2$

Solution:

$$\begin{aligned}
(2-i)+4+2i-i^2 &= 2-i+4+2i-(-1) && (28)\\
&= \boxed{7+i} && (29)
\end{aligned}$$

(b) Factor the following: $8x^3-1$

Solution:

$$\begin{aligned}
8x^3-1 &= (2x)^3-1^3 && (30)\\
&= \boxed{(2x-1)(4x^2+2x+1)} && (31)
\end{aligned}$$

5. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (12 pts)

(a) Solve for x: $2x^3 - 6x^2 = 36x$

Solution:

Our strategy is to get all terms on one side, a zero on the other side, and then factor.

$$\begin{aligned} 2x^3 - 6x^2 &= 36x && (32)\\ 2x^3 - 6x^2 - 36x &= 0 && (33)\\ 2x\left(x^2 - 3x - 18\right) &= 0 && (34)\\ 2x(x-6)(x+3) &= 0 && (35) \end{aligned}$$

So we get answers: $\boxed{x = 0, x = 6, x = -3}$.

(b) Solve for w: $3w - r = 2r - 5rw$

Solution:

The strategy here is the get all terms with a w on one side, all terms on the other side. Then factor and divide.

$$\begin{aligned} 3w - r &= 2r - 5rw && (36)\\ 3w + 5rw &= 2r + r && (37)\\ w(3 + 5r) &= 3r && (38)\\ w &= \boxed{\frac{3r}{3+5r}} && (39) \end{aligned}$$

(c) Solve for x: $\ln(x) - \ln(2) = \ln(1 - 3x)$

Solution:

We begin by using the laws of logarithms to rewrite the left hand side. Then we can use the one-to-one property for logarithms (if $\log_b(x_1) = \log_b(x_2)$ then $x_1 = x_2$) to solve.

$$\begin{aligned} \ln(x) - \ln(2) &= \ln(1 - 3x) && (40)\\ \ln\left(\frac{x}{2}\right) &= \ln(1 - 3x) && (41) \end{aligned}$$

Resulting in $\frac{x}{2} = 1 - 3x$. Solving this we get:

$$\begin{aligned} \frac{x}{2} &= 1 - 3x && (42)\\ \frac{x}{2} + 3x &= 1 && (43)\\ \frac{7x}{2} &= 1 && (44)\\ x &= \frac{2}{7} && (45) \end{aligned}$$

Checking this potential solution in the original equation we see it does indeed solve it so the answer is $\boxed{x = \frac{2}{7}}$.

6. For the two points $P(2, -3)$ and $Q(1, 5)$: (11 pts)

(a) Find the slope of the line through the two points.

Solution:

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - (-3)}{1 - 2} = \boxed{-8} \tag{46}$$

(b) Find the equation of the line that passes through the points P and Q.

Solution:

$$y - y_1 = m(x - x_1) \tag{47}$$
$$y - 5 = -8(x - 1) \tag{48}$$
$$y = -8x + 13 \tag{49}$$

So the answer is $\boxed{y = -8x + 13}$.

(c) Find the equation of the line through the point $R(0, 0)$ that is parallel to the line found in part (b).

Solution:

A line parallel to $y = -8x + 13$ has slope $m = -8$. The line that passes through $(0, 0)$ with slope $m = -8$ is:

$$y - y_1 = m(x - x_1) \tag{50}$$
$$y - 0 = -8(x - 0) \tag{51}$$
$$y = -8x \tag{52}$$

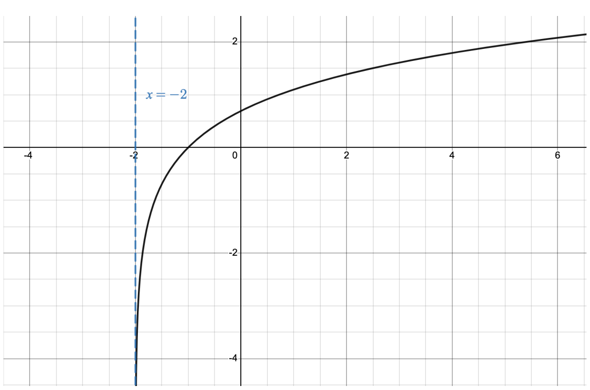
So the answer is $\boxed{y = -8x}$.

7. Consider the functions: $r(x) = \ln(x + 2)$ and $p(x) = e^x$. (8 pts)

(a) Find the domain of $r(x)$. Give your answer in interval notation.

Solution:

The domain of $r(x)$ includes values of x that make $\ln(x + 2)$ a real number. We can solve this by graphing $r(x) = \ln(x + 2)$.



From the graph we see that the domain is $\boxed{(-2, \infty)}$.

(b) Find $(p \circ r)(x)$.

Solution:

$(p \circ r)(x) = p(r(x)) = r(\ln(x+2)) = e^{\ln(x+2)} = \boxed{x+2}$.

(c) Find the domain of $(p \circ r)(x)$. Give your answer in interval notation.

Solution:

Since the domain of $r(x)$ is $(-2, \infty)$, the domain of $(p \circ r)(x)$ is restricted to $(-2, \infty)$. The domain of $p(x)$ is all real numbers, which does not restrict the domain of $(p \circ r)(x)$ any further, so the final answer is $\boxed{(-2, \infty)}$.

8. For the rational function $R(x) = \dfrac{x^2 - 4}{x^2 + 2x}$ answer the following (18 pts):

(a) Find the x and y-values of any hole(s). If there are none write NONE.

Solution:

We start by factoring $R(x)$:

$$R(x) = \frac{x^2 - 4}{x^2 + 2x} \tag{53}$$

$$= \frac{(x-2)(x+2)}{x(x+2)} \tag{54}$$

$$= \frac{x-2}{x} \tag{55}$$

So there is a hole when $x + 2 = 0$ at $x = -2$. The y-coordinate of the hole is found by plugging in $x = -2$ into the simplified fraction: $y = \dfrac{x-2}{x}$. We get $y = \dfrac{-2-2}{-2} = 0$. So the hole located at $\boxed{(-2, 2)}$.

(b) Find the x-intercept(s). If there are none write NONE.

Solution:

Setting $y = 0$ in the simplified rational function we get:

$$\frac{x-2}{x} = 0 \tag{56}$$

$$x - 2 = 0 \tag{57}$$

$$x = 2 \tag{58}$$

The only x-intercept is $\boxed{(2, 0)}$.

(c) Find the y-intercept. If there are none write NONE.

Solution:

Setting $x = 0$ in the simplified rational function, we get $y(0) = \dfrac{0-2}{0}$ which does not exist, so there is no y-intercept. $\boxed{NONE}$.

(d) Find the end behavior of the function.

Solution:

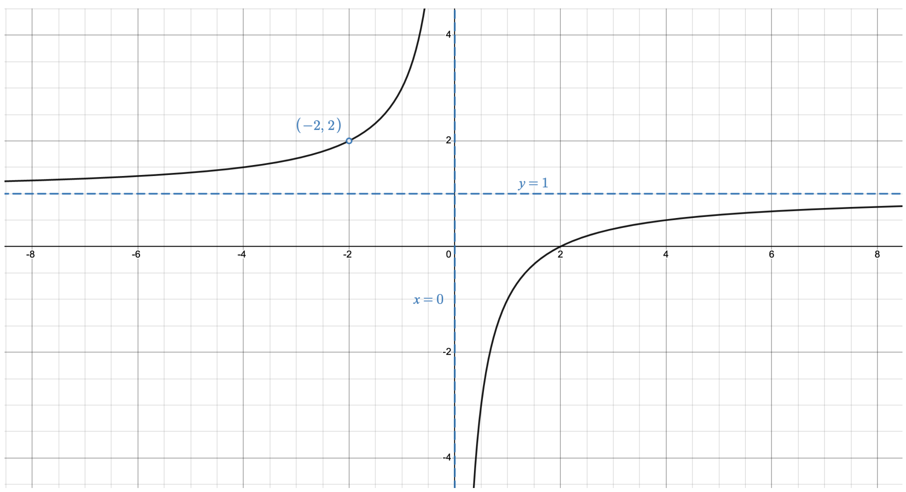
Using the simplified rational function: $y \to \dfrac{x}{x} = 1$ as $x \to \pm\infty$ so there is a horizontal asymptote of $\boxed{y = 1}$.

(e) Find all vertical asymptotes. If there are none write NONE.

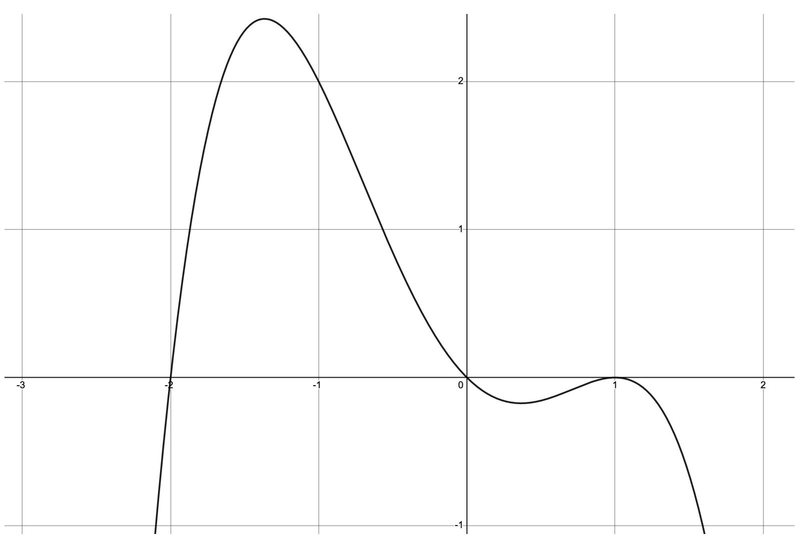
Solution:

Setting the denominator of the simplified rational function equal to zero we get one vertical asymptote: $\boxed{x = 0}$.

(f) Sketch the graph of $R(x)$ using your answers to parts a-e.



9. Write down a polynomial function, $P(x)$, that has the same intercepts, behavior at intercepts (cross or bounce), and end behavior as the graph below. You may leave your answer in factored form. (5 pts)



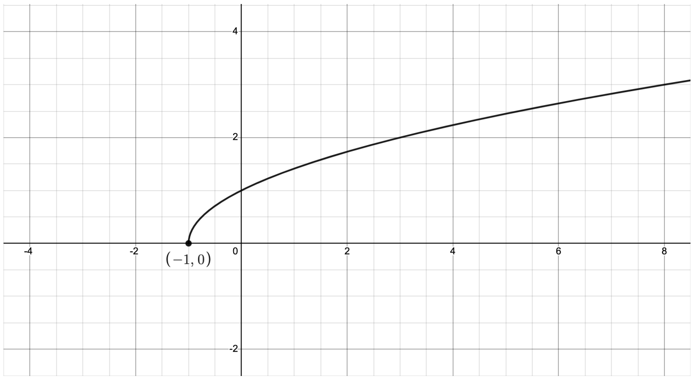
Solution:

The graph of $y = P(x)$ has zeros at $x = -2, 0, 1$ and crosses at $x = -2$ and $x = 1$ and bounces (does not cross) at $x = 1$. The polynomial should have odd multiplicity at $x = -2$ and $x = 1$ and even multiplicity at $x = 1$. So the polynomial has the form $y = x(x+2)(x-1)^2$. However, this has the wrong end behavior. The answer with the correct end behavior is: $\boxed{P(x) = -x(x+2)(x-1)^2}$.

10. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (13 pts)

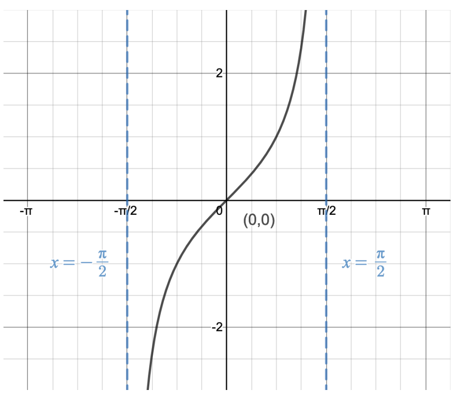
(a) $f(x) = \sqrt{x+1}$

Solution:



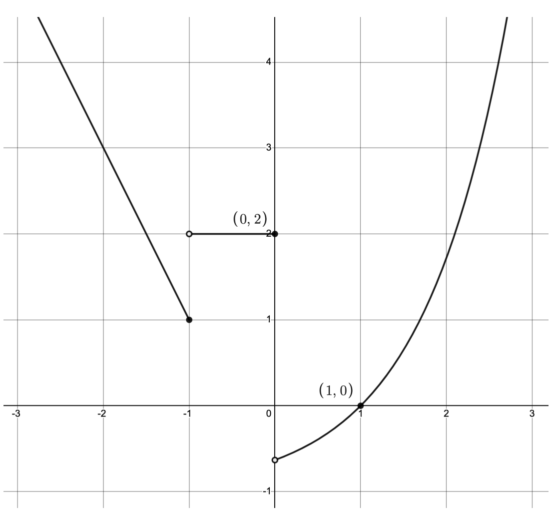
(b) $h(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Solution:



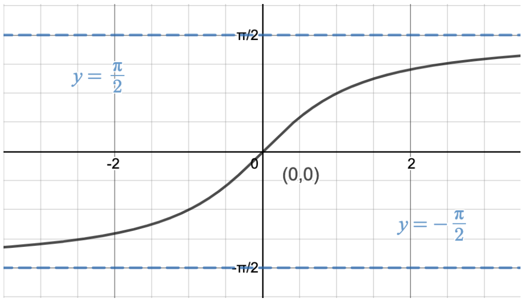
(c) $q(x) = \begin{cases} -2x - 1 & \text{if} \quad x \leq -1 \\ 2 & \text{if} \quad -1 < x \leq 0 \\ e^{x-1} - 1 & \text{if} \quad x > 0 \end{cases}$

Solution:



(d) $k(x) = \tan^{-1}(x)$

Solution:



11. Find the exact value: (15 pts)

(a) $\cos\left(\frac{3\pi}{4}\right)$

Solution:

$$\cos\left(\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$$

(b) $\sin\left(-\frac{2\pi}{3}\right)$

Solution:

$$\sin\left(-\frac{2\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$$

(c) $\sin^{-1}\left(\frac{1}{2}\right)$

Solution:

$$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$$

(d) $\arctan\left(\sqrt{3}\right)$

Solution:

$$\arctan\left(\sqrt{3}\right) = \boxed{\frac{\pi}{3}}$$

(e) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$

Solution:

$$\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \boxed{\frac{\pi}{6}}$$

(f) $\sin\left(\frac{\pi}{12}\right)$

Solution:

$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) =$ $\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$.

12. Verify the identity: $\csc(\theta) = \cos^2(\theta)\csc(\theta) + \sin(\theta)$. (5 pts)

This is one possible solution where we utilize $\cos^2(\theta) + \sin^2(\theta) = 1$:

$$\begin{aligned}
LHS &= \cos^2(\theta)\csc(\theta) + \sin(\theta) && (59)\\
&= \frac{\cos^2(\theta)}{\sin(\theta)} + \sin(\theta) && (60)\\
&= \frac{\cos^2(\theta)}{\sin(\theta)} + \frac{\sin^2(\theta)}{\sin(\theta)} && (61)\\
&= \frac{\cos^2(\theta) + \sin^2(\theta)}{\sin(\theta)} && (62)\\
&= \frac{1}{\sin(\theta)} && (63)\\
&= \csc(\theta) && (64)\\
&= RHS && (65)
\end{aligned}$$

13. Find all solutions to the following equations: (8 pts)

(a) $2\cos(\theta)\sin(\theta) - \cos(\theta) = 0$

Solution:

By factoring we get:

$$\begin{aligned}
2\cos(\theta)\sin(\theta) - \cos(\theta) &= 0 && (66)\\
\cos(\theta)\left(2\sin(\theta) - 1\right) &= 0 && (67)
\end{aligned}$$

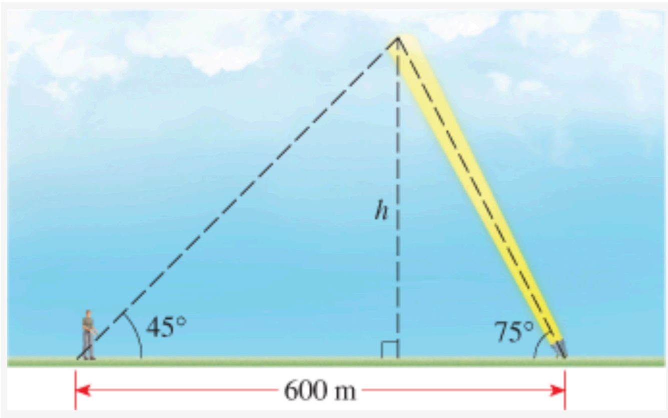
and by using the multiplicative property of zero we get $\cos(\theta) = 0$ and $2\sin(\theta) - 1 = 0$. $\cos(\theta) = 0$ has solutions $\theta = \frac{\pi}{2} + k2\pi$ where k is any integer. $2\sin(\theta) - 1 = 0$ has solutions when $\sin(\theta) = \frac{1}{2}$ which occurs when $\theta = \frac{\pi}{6} + k2\pi$ and $\theta = \frac{5\pi}{6} + k2\pi$. The resulting solutions of the original equation are: $\boxed{\theta = \frac{\pi}{2} + k2\pi}$, $\boxed{\theta = \frac{\pi}{6} + k2\pi}$, and $\boxed{\theta = \frac{5\pi}{6} + k2\pi}$.

(b) $\tan(4\theta) = 1$

Solution:

The equation, $\tan(4\theta) = 1$, is solved when $4\theta = \frac{\pi}{4} + k2\pi$ and $4\theta = \frac{5\pi}{4} + k2\pi$ where k is any integer. Dividing both sides by 4 we find the solutions of the original equation are: $\boxed{\theta = \frac{\pi}{16} + \frac{k\pi}{2}}$ and $\boxed{\theta = \frac{5\pi}{16} + \frac{k\pi}{2}}$.

14. To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle $75°$ from the horizontal. An observer 600m away measures the angle of elevation to the spot of light to be $45°$ (see image below). Find the height of the cloud cover. (6 pts)



Solution:

Starting with the right triangle involving the angle $45°$ and letting the length of the base from the person to the vertical line be y we can write down: $\tan(45°) = \frac{h}{y}$. The second triangle gives us $\tan(75°) = \frac{h}{600-y}$. Since $\tan(45°) = 1$, the first equation simplifies as $\frac{h}{y} = 1$ and $h = y$. Substituting this into the second equation, we can solve for h:

$$\tan(75°) = \frac{h}{600-h} \tag{68}$$
$$(600-h)\tan(75°) = h \tag{69}$$
$$600\tan(75°) - h\tan(75°) = h \tag{70}$$
$$600\tan(75°) = h + h\tan(75°) \tag{71}$$
$$600\tan(75°) = h(1+\tan(75°)) \tag{72}$$
$$h = \frac{600\tan(75°)}{1+\tan(75°)} \tag{73}$$

So the answer is $\boxed{h = \frac{600\tan(75°)}{1+\tan(75°)}\text{m}}$.

15. For $h(x) = 3\cos(2x)$ (12 pts)

(a) Identify the amplitude.

Solution:

Amplitude is $|a| = \boxed{3}$.

(b) Identify the period.

Solution:

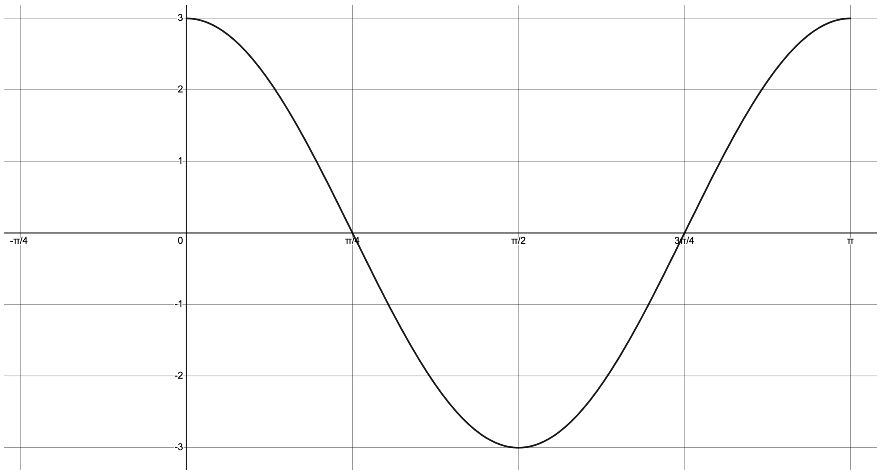
Period is $\frac{2\pi}{|b|} = \frac{2\pi}{|2|} = \boxed{\pi}$.

(c) Identify the phase shift.

Solution:

Phase shift is $-\frac{c}{b} = -\frac{0}{2} = \boxed{0}$.

(d) Sketch one cycle of the graph of $h(x)$. Be sure to label at least two values on the x axis and clearly identify the amplitude.



(e) Is $h(x)$ odd, even, or neither? Be sure to justify your answer for full credit.

Solution:

Since the graph is periodic, repeating its shape every period π, then the graph on $[-\pi, 0]$ has the same shape as that on $[0, \pi]$ and is symmetric about the y-axis and is $\boxed{\text{even}}$. This can be seen by graphing $h(x)$ on the restricted domain $[-\pi, \pi]$.