APPM 1235

INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources), help from another person, are not permitted during the exam. **Give all answers in exact form.**

11. Area of a sector: $A = \frac{1}{2}r^2\theta$

14. $\sin(2\theta) = 2\sin\theta\cos\theta$

15. $\cos(2\theta) = 1 - 2\sin^2\theta$

12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$

13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$

Potentially useful formulas:

- 1. $a^3 b^3 = (a b)(a^2 + ab + b^2)$ 10. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- 2. Circle: $(x h)^2 + (y k)^2 = r^2$
- 3. Arc length: $s = r\theta$
- 4. $\sin(a-b) = \sin a \cos b \sin b \cos a$
- 5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
- 6. $\cos(2\theta) = \cos^2\theta \sin^2\theta$
- 7. $\cos(2\theta) = 2\cos^2\theta 1$
- 8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$ 9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ 16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ 17. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

NOTE: YOU MAY TEAR OFF THIS PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

- i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.
- ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE
- iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE.
- iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND EXIT TO THE EXAM SUBMISSION AREA.

Final Exam

Scratch Paper

Name: _____

- 1. The following are unrelated. (7 pts)
 - (a) Place the correct symbol, <, >, or, = in the space between each of the following pair of numbers.

i.
$$\frac{2}{7}$$
 $\frac{8}{28}$

ii.
$$\sqrt{3}$$
 $\sqrt{2}$

- (b) Let a and b be real numbers such that a < 0 and b < 0. Determine whether the following expression is positive, negative, or zero: $-4a^4b^3$.
- (c) **True or False**: $\sqrt{x^2 + 4} = x + 2$. If false, pick a value for x and show, for this value, that the left side does not equal the right side.

2. The following are unrelated. Leave answers without negative exponents. (8 pts)

(a) Evaluate and simplify:
$$-\frac{1}{15} + \frac{3}{10} - \frac{1}{\frac{1}{3}} - 4^{-1/2} + \sqrt{9}$$

(b) Simplify:
$$-\frac{3x^{-2}}{x^{-3}} + 2(1-x)(x+3)$$

3. The following are unrelated. (16 pts)

(a) Subtract and simplify:
$$\frac{x+1}{x^2+4x+4} - \frac{4}{x^2-2x}$$

(b) Simplify:
$$\frac{\sqrt{r^8}}{(16r^2)^{1/4}}$$

(c) Simplify:
$$\frac{\frac{2}{x+1} - 1}{\frac{2}{x^2} - \frac{2}{x}}$$

(d) Simplify the following: $\ln(1) - \ln(e) + \log(100) - \log_3\left(\frac{1}{9}\right)$ (Your answer should have no logarithms)

- 4. The following are unrelated: (6 pts)
 - (a) Simplify and write in a + bi form: $(2 i) + 4 + 2i i^2$

(b) Factor the following: $8x^3 - 1$

5. Solve the following equations for the indicated variable. If there are no solutions, write no solutions. (12 pts)

(a) Solve for *x*: $2x^3 - 6x^2 = 36x$

(b) Solve for w: 3w - r = 2r - 5rw

(c) Solve for x: $\ln(x) - \ln(2) = \ln(1 - 3x)$

- 6. For the two points P(2, -3) and Q(1, 5): (11 pts)
 - (a) Find the slope of the line through the two points.
 - (b) Find the equation of the line that passes through the points P and Q.

(c) Find the equation of the line through the point R(0,0) that is parallel to the line found in part (b).

- 7. Consider the functions: $r(x) = \ln(x+2)$ and $p(x) = e^x$. (8 pts)
 - (a) Find the domain of r(x). Give your answer in interval notation.
 - (b) Find $(p \circ r)(x)$.
 - (c) Find the domain of $(p \circ r)(x)$. Give your answer in interval notation.

- 8. For the rational function $R(x) = \frac{x^2 4}{x^2 + 2x}$ answer the following (18 pts):
 - (a) Find the x and y-values of any hole(s). If there are none write NONE.

- (b) Find the *x*-intercept(s). If there are none write NONE.
- (c) Find the *y*-intercept. If there are none write NONE.
- (d) Find the end behavior of the function.
- (e) Find all vertical asymptotes. If there are none write NONE.
- (f) Sketch the graph of R(x) using your answers to parts a-e.



9. Write down a polynomial function, P(x), that has the same intercepts, behavior at intercepts (cross or bounce), and end behavior as the graph below. You may leave your answer in factored form. (5 pts)



10. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (13 pts)









(c)	q(x	r) =	-2x 2 e^{x-1}	— 1 — 1	i i l	if if	$x \le -1$ $x >$	-1 < x 0	$c \leq 0$
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11. Find the exact value: (15 pts)

(a)
$$\cos\left(\frac{3\pi}{4}\right)$$
 (d) $\arctan\left(\sqrt{3}\right)$

(b)
$$\sin\left(-\frac{2\pi}{3}\right)$$
 (e) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$

(c)
$$\sin^{-1}\left(\frac{1}{2}\right)$$
 (f) $\sin\left(\frac{\pi}{12}\right)$

12. Verify the identity: $\csc(\theta) = \cos^2(\theta) \csc(\theta) + \sin(\theta)$. (5 pts)

13. Find all solutions to the following equations: (8 pts)

(a) $2\cos(\theta)\sin(\theta) - \cos(\theta) = 0$

(b) $\tan(4\theta) = 1$

14. To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle 75° from the horizontal. An observer 600m away measures the angle of elevation to the spot of light to be 45° (see image below). Find the height of the cloud cover. (6 pts)



- 15. For $h(x) = 3\cos(2x)$ (12 pts)
 - (a) Identify the amplitude.
 - (b) Identify the period.
 - (c) Identify the phase shift.
 - (d) Sketch one cycle of the graph of h(x). Be sure to label at least two values on the *x*-axis and clearly identify the amplitude.



(e) Is h(x) odd, even, or neither? Be sure to justify your answer for full credit.