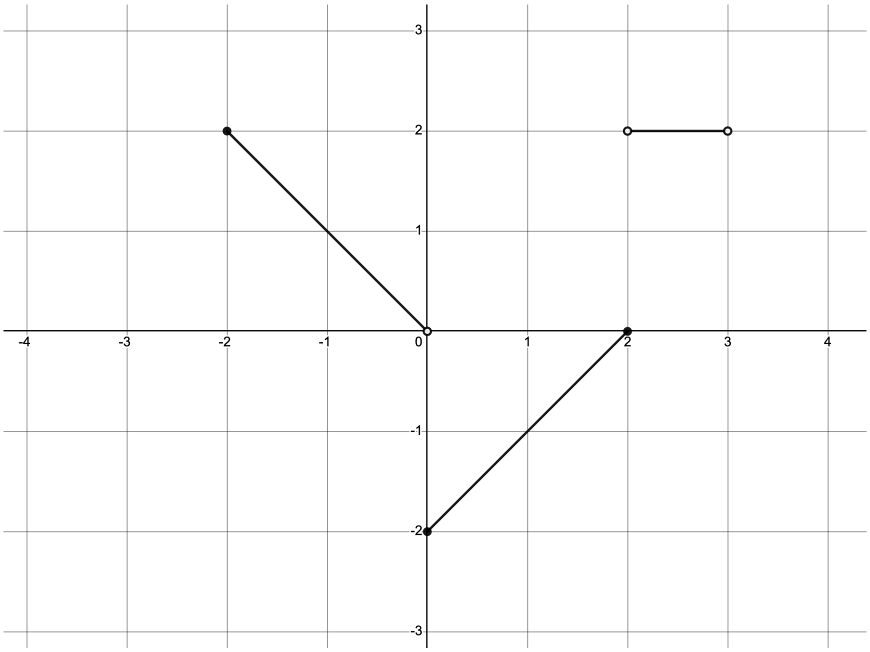
1. Answer the following for the given graph of a function f (no justification is needed for this problem) (11pts):



(a) Identify the domain of f.

Solution:

$\boxed{[-2, 3)}$

(b) Identify the the range of f.

Solution:

$\boxed{[-2, 2]}$

(c) Find $f(0)$ if it exists.

Solution:

$f(0) = \boxed{-2}$

(d) Find $(f \cdot f)(1)$

Solution:

$(f \cdot f)(1) = f(1) \cdot f(1) = (-1)(-1) = \boxed{1}$

(e) Is f odd, even, or neither?

Solution:

The graph of f is neither symmetric about the y-axis nor the origin so it is $\boxed{neither}$ odd or even.

(f) Find $(f \circ f)(2.5)$.

Solution:

$(f \circ f)(2.5) = f(f(2.5)) = f(2) = \boxed{0}$

(g) f is not one-to-one. Briefly explain why it isn't one-to-one.

Solution:

$\boxed{\text{The graph of } f \text{ does not pass the horizontal line test}}$. For example, $f(-2) = 2$ and $f(2.5) = 2$ but $-2 \neq 2.5$.

(h) Identify a restriction of the domain so that f is one-to-one and has the same range as in part (b).

Solution:

$\boxed{[-2, 2]}$

(i) Use your domain restriction to calculate $f^{-1}(2)$.

Solution:

$f^{-1}(2) = \boxed{-2}$

(j) Find the x-values where $f(x) < 0$. Give your answer in interval notation.

Solution:

$\boxed{[0, 2)}$

(k) Find the average rate of change from $x = -1$ to $x = 0$.

Solution:

$$\frac{f(-1) - f(0)}{-1 - 0} = \frac{1 - (-2)}{-1} = \boxed{-3}$$

2. The following are unrelated: (9 pts)

   (a) Find the equation of the circle centered at $(-3, 1)$ with radius $r = \sqrt{5}$.

   Solution:

   $\boxed{(x+3)^2 + (y-1)^2 = 5}$

   (b) Find the equation of the line that crosses through the points $(1, -2)$ and $(4, 3)$.

   Solution:

   The slope equation gives us $m = \dfrac{3-(-2)}{4-1} = \dfrac{5}{3}$

   Using the slope-intercept equation, we get $y = \dfrac{5}{3}x + b$. To solve for b we substitute in the point $(4, 3)$.

$$
\begin{align}
y &= \frac{5}{3}x + b \tag{1}\\
3 &= \frac{5}{3}4 + b \tag{2}\\
9 &= 20 + 3b \tag{3}\\
-\frac{11}{3} &= b \tag{4}
\end{align}
$$

   So we get the equation of the line $\boxed{y = \dfrac{5}{3}x - \dfrac{11}{3}}$.

   (c) Find the equation of the vertical line that crosses through the point $(5, 7)$.

   Solution:

   $\boxed{x = 5}$

3. Suppose a is a constant and you are given two points $(-2, a)$ and $(-1, 1)$. Now suppose you know the midpoint between the two points is $\left(-\dfrac{3}{2}, \dfrac{5}{2}\right)$. Find the value for a. (3 pts)

   Solution:

   The y-coordinate for the midpoint is found by writing $\dfrac{a+1}{2} = \dfrac{5}{2}$. Now we can solve for a:

$$
\begin{align}
\frac{a+1}{2} &= \frac{5}{2} \tag{5}\\
a + 1 &= 5 \tag{6}\\
a &= \boxed{4} \tag{7}
\end{align}
$$

4. Find the domain of the following functions. Express your answers in interval notation. (15 pts)

(a) $m(x) = \dfrac{x-4}{x^2-16}$

Solution:

The domain is all real numbers except when $x^2 - 16 = 0$.

$$\begin{aligned} x^2 - 16 &= 0 && (8)\\ x^2 &= 16 && (9)\\ x &= \pm 4 && (10) \end{aligned}$$

So the domain is $\boxed{(-\infty, -4) \cup (-4, 4) \cup (4, \infty)}$.

(b) $q(r) = \dfrac{3\sqrt{r}}{r-1}$

Solution:

Since the square root of a negative number does not exist in the real numbers, then $r \geq 0$. However, $r - 1$ in the denominator cannot be zero, so $r \neq 1$. Thus the domain is $\boxed{[0, 1) \cup (1, \infty)}$.

(c) $r(t) = 5\sqrt[3]{t-1}$

Solution:

Since the cube root of a negative number results in a negative number, then the domain is $\boxed{(-\infty, \infty)}$.

5. For $f(x) = \sqrt{x-3}$ and $k(x) = x^2 + 3$, find the following: (10 pts)

(a) Find $f(11)$

Solution:

$f(11) = \sqrt{11-3} = \sqrt{8} = \boxed{2\sqrt{2}}$

(b) Find $f(x+12)$

Solution:

$f(x+12) = \sqrt{x+12-3} = \boxed{\sqrt{x+9}}$

(c) Find $(k \circ f)(x)$.

Solution:

$(k \circ f)(x) = k(f(x)) = k\left(\sqrt{x-3}\right) = \left(\sqrt{x-3}\right)^2 + 3 = x - 3 + 3 = \boxed{x}$.

(d) Find the domain of $(k \circ f)(x)$.

Solution:

The domain of $(k \circ f)(x)$ is $[3, \infty)$ since $\sqrt{x-3}$ requires $x - 3 \geq 0$.

6. Answer the following for the one-to-one function $g(x) = \dfrac{1}{x-2}$. (5 pts)

(a) Find $g^{-1}(x)$.

Solution:

First we let $g(x) = y = \dfrac{1}{x-2}$. Now we solve for x:

$$
\begin{aligned}
y &= \frac{1}{x-2} && (11) \\
y(x-2) &= 1 && (12) \\
x - 2 &= \frac{1}{y} && (13) \\
x &= \frac{1}{y} + 2 && (14)
\end{aligned}
$$

Swapping x and y and substituting the new y for $g^{-1}(x)$ we get $\boxed{g^{-1}(x) = \dfrac{1}{x} + 2}$.

(b) What is the range of $g^{-1}(x)$?

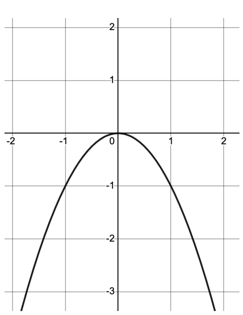
Solution:

The range of $g^{-1}(x)$ is the domain of $g(x) = \dfrac{1}{x-2}$ which is $\boxed{(-\infty, 2) \cup (2, \infty)}$.

7. Sketch the shape of the graph of each of the following on the provided axes. Make sure to label relevant value(s) on your axe(s) (19 pts)

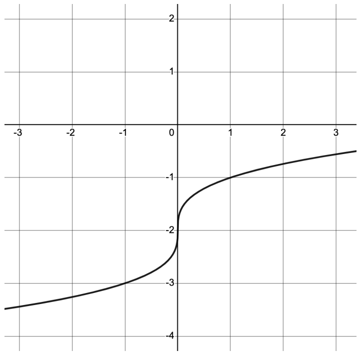
(a) $f(x) = -x^2$

Solution:



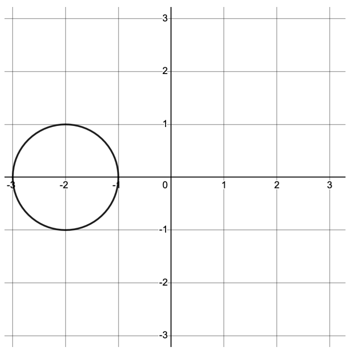
(b) $k(x) = \sqrt[3]{x} - 2$

Solution:



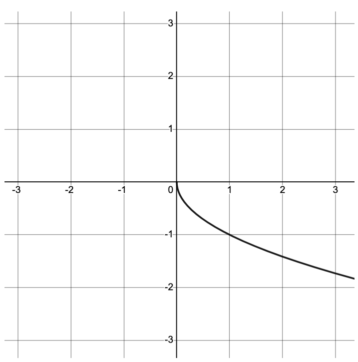
(c) $(x+2)^2 + y^2 = 1$

Solution:



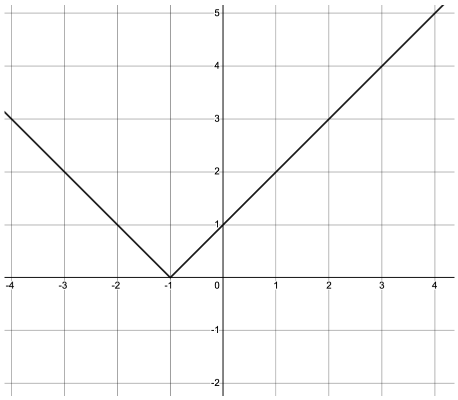
(d) $g(x) = -\sqrt{x}$

Solution:



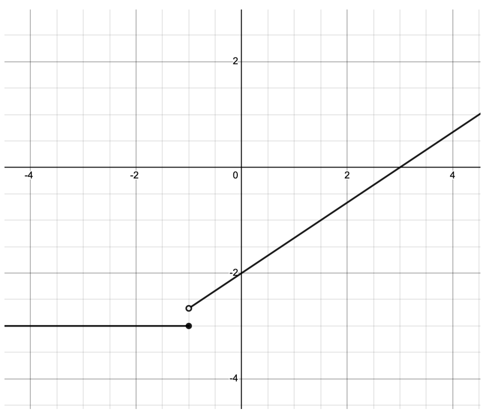
(e) $m(x) = |x+1|$

Solution:



(f) $q(x) = \begin{cases} -3 & \text{if} \quad x \le -1 \\ \frac{2}{3}x - 2 & \text{if} \quad x > -1 \end{cases}$

Solution:



8. For $P(x) = 2x^3 + 8x^2 + 8x$ answer the following. (7 pts)

(a) Indicate on a graph or use arrow notation to indicate the end behavior of $P(x)$.

Solution:

For end behavior: $P(x) \approx 2x^3 \to \infty$ as $x \to \infty$ and $P(x) \approx 2x^3 \to -\infty$ as $x \to -\infty$.

(b) Find the y-intercept of $P(x)$.

Solution:

The y-intercept is found by setting $x = 0$. So $P(0) = 2\left(0^3\right) + 8\left(0^2\right) + 8(0) = 0$. So the y-intercept is $\boxed{(0,0)}$.

(c) Find all zeros and identify the multiplicity of each zero.

Solution:

The zeros of a polynomial are the x-values that result in $P(x) = 0$. So we set $2x^3 + 8x^2 + 8x = 0$. By factoring:

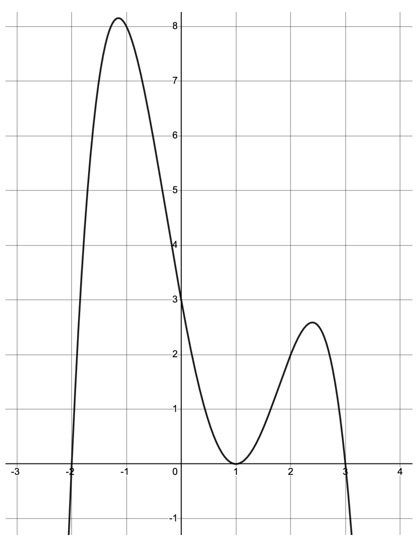
$$
\begin{aligned}
2x^3 + 8x^2 + 8x &= 0 && (15)\\
2x\left(x^2 + 4x + 4\right) &= 0 && (16)\\
2x(x+2)(x+2) &= 0 && (17)\\
2x(x+2)^2 &= 0 && (18)
\end{aligned}
$$

So we get $x = 0$ and $x = -2$ as the zeros. The multiplicity of $x = 0$ is $\boxed{1}$ and $x = -2$ is $\boxed{2}$.

9. Sketch the shape of the graph of a polynomial function, $g(x)$, that satisfies **all** of the information. **Label** all intercepts on the graph. (5 pts)

   i. The graph has y-intercept $(0, 3)$.
   ii. The graph has end behavior consistent with $y = -\frac{1}{2}x^4$.
   iii. The graph crosses at $(-2, 0)$ and $(3, 0)$ and bounces (touches but does not cross) at $(1, 0)$.
   iv. The graph has no other x-intercepts.

Solution:



10. Use long division to find the quotient and remainder when $x^3 - 2x^2 + x - 7$ is divided by $x - 2$. (4 pts)

Solution:

$$
\begin{array}{rrrrr}
 & x^2 + & & 1 & \\ \cline{2-5}
x - 2 \;\big) & x^3 - & 2x^2 + & x - & 7 \\
 & -(x^3 - & 2x^2) & & \\ \cline{2-3}
 & & 0 + & x - & 7 \\
 & & & -(x - & 2) \\ \cline{3-5}
 & & & & -5
\end{array}
$$

So the quotient is $x^2 + 1$ and the remainder is -5.

11. The following are unrelated.

(a) Is $f(x) = x^3 - 2x$ odd, even, or neither? Justify your answer for full credit. (4 pts)

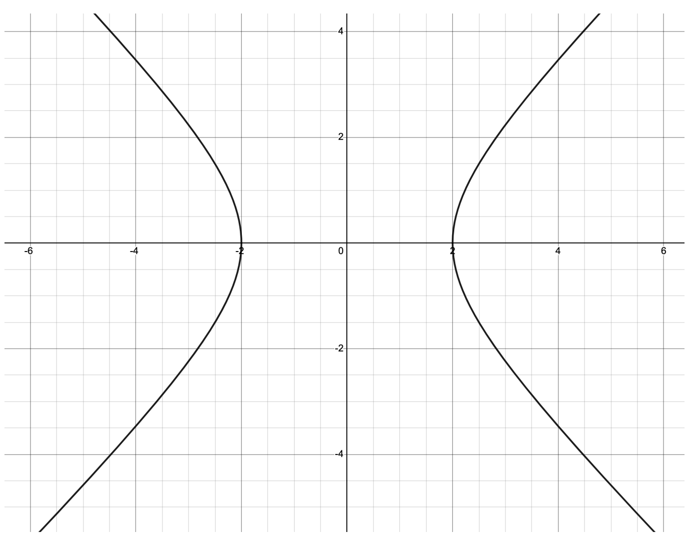
Solution:

To determine if $f(x) = x^3 - 2x$ is odd, even, or neither, we substitute x for $-x$ and see if this results in $f(x)$ or $-f(x)$ or neither.

$f(-x) = (-x)^3 - 2(-x) = -x^3 + 2x = -\left(x^3 - 2x\right) = -f(x)$

So $f(x)$ is $\boxed{\text{odd}}$.

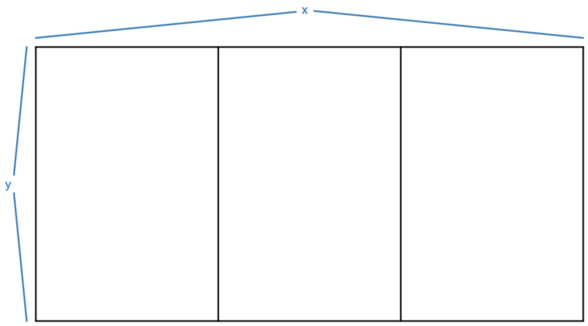
(b) Is the graph below that of a function? Briefly explain why or why not. (3 pts)



Solution:

This is not the graph of a function since it does not pass the vertical line test. In other words: $y(4) = \pm 3.5$ means this cannot be the graph of a function.

12. A farmer uses 6 kilometers of fencing to fence in three equal-sized rectangular plots of land (see image below). Find the dimensions of each rectangle that maximizes the total area of all three rectangles. (5 pts)



Solution:

Step 1: Draw picture is done for us.

Step 2: Identify what we are trying to maximize and give it a variable. Let A represent the total area. Considering all rectangles together, we can label one side y and the other side x (as denoted in the picture)

Step 3: Identify first equation: The total area is $A = xy$.

Step 4: There are too many variables to work with so we identify a second equation. We used six kilometers of fencing to create the pens, so we know that $2x + 4y = 6$.

Step 5: Now we get the equation for area with just one other variable. Solving $2x + 4y = 6$ for x we get $x = 3 - 2y$. By substitution, we get an equation for the area: $A = (3 - 2y)y = -2y^2 + 3y$.

Step 6: Answer the question: The graph of $A = -2y^2 + 3y$ is a downward pointing parabola, so there is a maximum by using the vertex formula: $y = \frac{-b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4}$ km. We can find x by using the fact that $x = 3 - 2y = 3 - 2\left(\frac{3}{4}\right) = 3 - \frac{3}{2} = \frac{3}{2}$ km. However, the problem asks for the dimensions of each individual rectangle. Since there are three rectangles, we need to divide $x = \frac{3}{2}$ by 3 to get the width of each individual rectangle as $x = \frac{1}{2}$ km, so we get the answer $\frac{1}{2}$ km $\times$ $\frac{3}{4}$ km.