1. The following are unrelated: (20 pts)

   (a) Use the distributive property to rewrite the expression (you do not need to carry out any mathematical operations):

   $$3\,(x+4)^5\,(x+1)^3 - (x-2)^4\,(4)\,(x+1)^3$$

   Solution:

   $$3\,(x+4)^5\,(x+1)^3 - (x-2)^4\,(4)\,(x+1)^3 = \boxed{(x+1)^3\left[3\,(x+4)^5 - (x-2)^4\,(4)\right]}$$

   (b) Add/Subtract as indicated: $-\dfrac{5}{12} - \dfrac{1}{4} + \dfrac{5}{6^1} + 3\sqrt{2} + \sqrt{2}$

   Solution:

   $$-\frac{5}{12} - \frac{1}{4} + \frac{5}{6^1} + 3\sqrt{2} + \sqrt{2} = -\frac{5}{12} - \frac{3}{12} + \frac{10}{12} + 3\sqrt{2} + \sqrt{2} = \frac{2}{12} + 4\sqrt{2} = \boxed{\frac{1}{6} + 4\sqrt{2}}.$$

   (c) Let a and b be real numbers such that $a < 0$ and $b > 0$. Answer the following:

      i. Is $-a + b$ positive or negative? No work is needed to justify your answer.

      Solution:

      $\boxed{\text{Positive}}$

      ii. Is a^{18} positive or negative? No work is needed to justify your answer.

      Solution:

      $\boxed{\text{Positive}}$

   (d) Multiply as indicated (give answer in $a + bi$ form): $\left(\dfrac{1}{2}i\right)\left(-2 + \dfrac{1}{2}i\right)$

   Solution:

   $$\left(\frac{1}{2}i\right)\left(-2 + \frac{1}{2}i\right) = -i + \frac{1}{4}i^2 = -i - \frac{1}{4} = \boxed{-\frac{1}{4} - i}$$

   (e) Simplify: $2 + (3-x)(2x+1) + x^2$.

   Solution:

   $$2 + (3-x)(2x+1) + x^2 = 2 + 6x + 3 - 2x^2 - x + x^2 = \boxed{5 + 5x - x^2}$$

2. The following are unrelated: (24 pts)

(a) Evaluate the expression: $\sqrt{24}$

Solution:

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = \boxed{2\sqrt{6}}$$

(b) Evaluate the expression: $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}}$

Solution:

$$\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = \boxed{2}$$

(c) Rewrite with only rational exponents (you do not need to carry out any mathematical operations, your answer should not have any roots):

$$\sqrt[4]{3} + \sqrt{x^2 - 9}$$

Solution:

$$\sqrt[4]{3} + \sqrt{x^2 - 9} = \boxed{3^{1/4} + \left(x^2 - 9\right)^{1/2}}$$

(d) Simplify (Give your answer without negative exponents): $\left(6w^{-2}z^3\right)^2 \left(2z^3\right)$

Solution:

$$\begin{aligned}\left(6w^{-2}z^3\right)^2 \left(2z^3\right) &= \left(36w^{-4}z^6\right)\left(2z^3\right)\\ &= \boxed{\frac{72z^9}{w^4}}\end{aligned}$$

(e) Simplify (Give your answer without negative exponents): $4x^{1/2}\left(x^{1/2} - x^{-1/4}\right)$

Solution:

$$\begin{aligned}4x^{1/2}\left(x^{1/2} - x^{-1/4}\right) &= 4x^{1/2}x^{1/2} - 4x^{1/2}x^{-1/4}\\ &= 4x^{1/2+1/2} - 4x^{1/2-1/4}\\ &= \boxed{4x - 4x^{1/4}}\end{aligned}$$

(f) Multiply: $\left(4x^4+\dfrac{1}{x^4}\right)^2$

Solution:

$$\begin{aligned}\left(4x^4+\frac{1}{x^4}\right)^2 &= \left(4x^4+\frac{1}{x^4}\right)\left(4x^4+\frac{1}{x^4}\right)\\ &= \left(4x^4\right)\left(4x^4\right)+\left(4x^4\right)\left(\frac{1}{x^4}\right)+\left(\frac{1}{x^4}\right)\left(4x^4\right)+\left(\frac{1}{x^4}\right)\left(\frac{1}{x^4}\right)\\ &= 16x^8+4+4+\frac{1}{x^8}\\ &= \boxed{16x^8+8+\frac{1}{x^8}}\end{aligned}$$

3. The following are unrelated: (20 pts)

(a) Factor completely (If not factorable write NF): $2x^2-12x+16$

Solution:

$$\begin{aligned}2x^2-12x+16 &= 2\left(x^2-6x+8\right)\\ &= \boxed{2(x-4)(x-2)}\end{aligned}$$

(b) Evaluate $x^2+16-\sqrt{2x}$ for $x=2$

Solution:

$$\begin{aligned}2^2+16-\sqrt{2\cdot 2} &= 4+16-2\\ &= \boxed{18}\end{aligned}$$

(c) Factor completely (Hint, factor out the lowest power of x): $x^{5/2}-x^{1/2}$

Solution:

$$\begin{aligned}x^{5/2}-x^{1/2} &= x^{1/2}\left(x^{5/2-1/2}-1\right)\\ &= x^{1/2}\left(x^2-1\right)\\ &= \boxed{x^{1/2}\left(x-1\right)\left(x+1\right)}\end{aligned}$$

(d) Simplify the complex fraction: $\dfrac{-2+\frac{4}{x+1}}{1-\frac{1}{x}}$

Solution:

$$
\begin{aligned}
\frac{-2+\frac{4}{x+1}}{1-\frac{1}{x}} &= \frac{-2\frac{x+1}{x+1}+\frac{4}{x+1}}{\frac{x}{x}-\frac{1}{x}} \\
&= \frac{\frac{-2x-2+4}{x+1}}{\frac{x-1}{x}} \\
&= \frac{-2x+2}{x+1}\cdot\frac{x}{x-1} \\
&= \frac{-2(x-1)}{x+1}\cdot\frac{x}{x-1} \\
&= \boxed{\frac{-2x}{x+1}}
\end{aligned}
$$

(e) Subtract: $\dfrac{1}{4x+8}-\dfrac{x+3}{x^2-4}$

Solution:

$$
\begin{aligned}
\frac{1}{4x+8}-\frac{x+1}{x^3-4x} &= \frac{1}{4(x+2)}-\frac{x+3}{(x-2)(x+2)} \\
&= \frac{x-2}{4(x-2)(x+2)}-\frac{4(x+3)}{4(x-2)(x+2)} \\
&= \frac{x-2-4(x+3)}{4(x-2)(x+2)} \\
&= \boxed{\frac{-3x-14}{4(x-2)(x+2)}}
\end{aligned}
$$

4. Suppose you know that b is a constant and that the equation $x^2+bx+2=0$ has a solution of $x=4$. Find b. (4 pts)

Solution:

Plugging in $x=4$ we can solve: $4^2+b(4)+2=0$ for b.

$$
\begin{aligned}
4^2+b(4)+2&=0 \\
16+4b+2&=0 \\
4b&=-18 \\
b&=-\frac{18}{4}=\boxed{-\frac{9}{2}}
\end{aligned}
$$

5. Solve each of the following equations: (12 pts)

(a) $2 - \frac{3}{4}x = \frac{1}{2}x$

Solution:

We start by clearing the fractions by multiplying both sides of the equation by 4.

$$\begin{aligned}
2 - \frac{3}{4}x &= \frac{1}{2}x \\
4\left(2 - \frac{3}{4}x\right) &= 4\left(\frac{1}{2}x\right) \\
4(2) - 4\left(\frac{3}{4}x\right) &= 4\left(\frac{1}{2}x\right) \\
8 - 3x &= 2x \\
8 &= 5x \\
x &= \boxed{\frac{8}{5}}
\end{aligned}$$

(b) $x^2 = -3x - 2$

Solution:

We start by moving all terms to one side and then factoring.

$$\begin{aligned}
x^2 &= -3x - 2 \\
x^2 + 3x + 2 &= 0 \\
(x+2)(x+1) &= 0
\end{aligned}$$

By the multiplicative property of zero, we set $x + 2 = 0$ and $x + 1 = 0$ resulting in solutions $\boxed{x = -2}$ and $\boxed{x = -1}$.

(c) $2 + \sqrt{x} = x$

Solution:

We start by isolating the square root so we can square both sides. We then can solve the equation by factoring.

$$\begin{aligned}
2 + \sqrt{x} &= x \\
\sqrt{x} &= x - 2 \\
x &= (x-2)^2 \\
x &= x^2 - 4x + 4 \\
0 &= x^2 - 5x + 4 \\
0 &= (x-4)(x-1)
\end{aligned}$$

This results in two potential solutions: $x = 4$ and $x = 1$. We must check both values:

Checking $x = 4$ we get $2 + \sqrt{4} = 2 + 2 = 4$ which is a solution.

Checking $x = 1$ we get $2 + \sqrt{1} = 2 + 1 = 3 \neq 1$ which is not a solution.

So the only solution is: $\boxed{x = 4}$.

6. Solve each of the following equations: (12 pts)

(a) Solve for x: $T^2 = 2a(x - z)$

Solution:

$$\begin{aligned}
T^2 &= 2a(x - z) \\
\frac{T^2}{2a} &= x - z \\
\frac{T^2}{2a} + z &= x
\end{aligned}$$

So the answer is $\boxed{x = \frac{T^2}{2a} + z}$.

(b) $\dfrac{1}{x^2+1} = \dfrac{4x+1}{(x^2+1)^2}$

Solution:

$$\begin{aligned}
\frac{1}{x^2+1} &= \frac{4x+1}{(x^2+1)^2} \\
\left(x^2+1\right)^2 \cdot \frac{1}{x^2+1} &= \left(x^2+1\right)^2 \cdot \frac{4x+1}{(x^2+1)^2} \\
x^2+1 &= 4x+1 \\
x^2-4x &= 0 \\
x(x-4) &= 0
\end{aligned}$$

Resulting in two potential solutions: $x = 0$ and $x = 4$. Plugging in $\boxed{x=0}$ and $\boxed{x=4}$ we see that both are in fact solutions.

(c) $|x-3| = 7$

Solution:

The x-values that solve $|x-3| = 7$ are found when we set $x-3 = -7$ and $x-3 = 7$. This results in two solutions: $\boxed{x=-4}$ and $\boxed{x=10}$.

7. Solve the following inequalities. Justify your answers by using a number line or sign chart. Answers without full justification will not receive full credit. Express all answers in interval notation. (12 pts)

(a) $1-4x < 3$

Solution:

$$\begin{aligned}
1-4x &< 3 \\
-4x &< 2 \\
x &> -\frac{2}{4} \\
x &> -\frac{1}{2}
\end{aligned}$$

The solution in interval notation is $\boxed{\left(-\frac{1}{2}, \infty\right)}$.

(b) $x^3 + 2x^2 \geq 0$

Solution:

$$x^3 + 2x^2 \geq 0$$
$$x^2\left(x+2\right) \geq 0$$

This results in two values that make the left side zero: $x = 0$ and $x = -2$.

Placing these on a number line and picking test values we get the solution $\boxed{[-2, \infty)}$.