INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources) are not permitted. **Give all answers in exact form.**

Potentially useful formulas:

1. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND THE NEXT PAGE AND USE (FRONT AND BACK) OF BOTH AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE

iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE. JUST BEFORE YOU UPLOAD TO GRADESCOPE WRITE DOWN YOUR UPLOAD TIME ON THE FIRST EXAM PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND EXIT TO THE EXAM SUBMISSION AREA.

Scratch Paper

Scratch Paper

Scratch Paper

Name: _____ Upload time: _____

1. The following are unrelated. Simplify answers and leave without negative exponents. (28 pts)

   (a) Simplify: $5x(x - x^3) - (-3x^2 - 1)^2$

   (b) Simplify: $(-xy^3)^2 \left(\frac{x^{-5}}{y^{-2}}\right) + 3\frac{y^8}{x^3}$

   (c) Add: $\frac{x}{x^2 - 1} + \frac{1}{x^2 + x}$

   (d) Multiply: $(24i - 8)(4i + 1)$. Give answer in $a + bi$ form.

(e) Multiply: $\left(x^{1/2} - y^{1/3}\right)\left(x^{1/2} + y^{1/3}\right)$

(f) Simplify: $\dfrac{\frac{1}{x-2} + 1}{\frac{1}{x-2}}$

(g) Simplify: $3^{4\log_3(x)} + \log_8(4) + \log_8(2) - \ln\left(e^2\right) + \log_2(32)$ (Your answer should have no logarithms)

2. Solve the following equations for x: (16 pts)

(a) $\frac{1}{3}x - \frac{3}{4} = \frac{1}{2}x$

(b) $\sqrt{6} - 3x^2 = x^2$

(c) $\log_2(1-x) - \log_2(x) = 2$

(d) $6 = 2e^{-x-1}$

3. For $g(x) = \dfrac{x^3 + 5x^2 + 6x}{x^2 - 9}$ answer the following (16 pts):

(a) Find the domain of $g(x)$. Give your answer in interval notation.

(b) Find the x, y-coordinates for any hole(s). If there are none write NONE.

(c) Find any horizontal or slant asymptotes. If there are none write NONE.

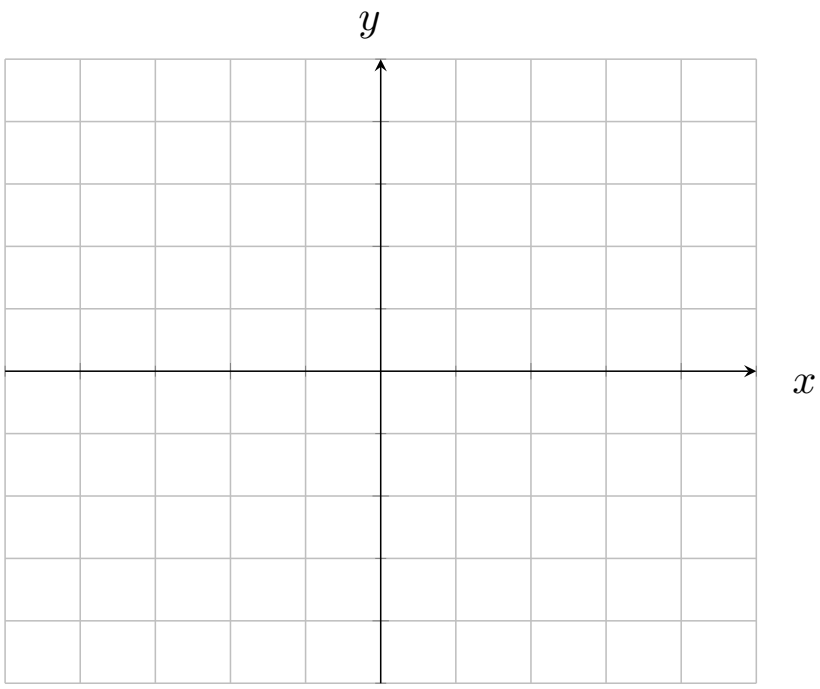
(d) Find any vertical asymptotes. If there are none write NONE.

4. Answer the following for the polynomial $P(x)$. (8 pts)

(a) Sketch the graph of $y = P(x)$ with the following properties:

i. $y \to \infty$ as $x \to -\infty$ and $y \to -\infty$ as $x \to \infty$.

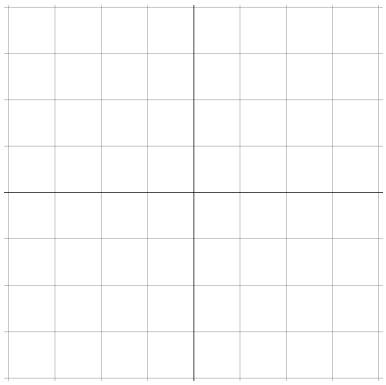
ii. The graph crosses the x-axis at $x = -1$ and touches, but does not cross, the x-axis at $x = 2$. The graph has y-intercept of $(0, -3)$. The graph has no other intercepts.



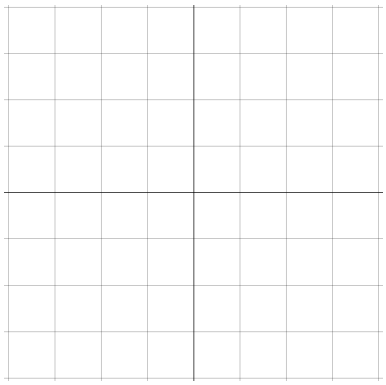
(b) Write down a polynomial that satisfies the given information.

5. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (13 pts)

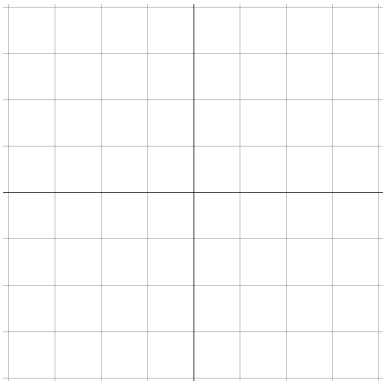
(a) $f(x) = x^2 - 1$.



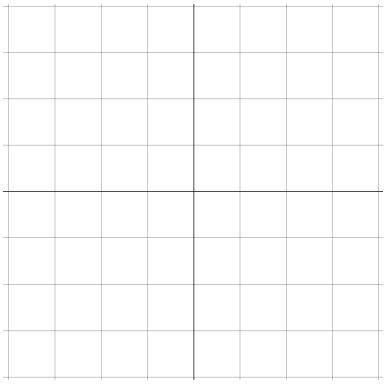
(b) $h(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



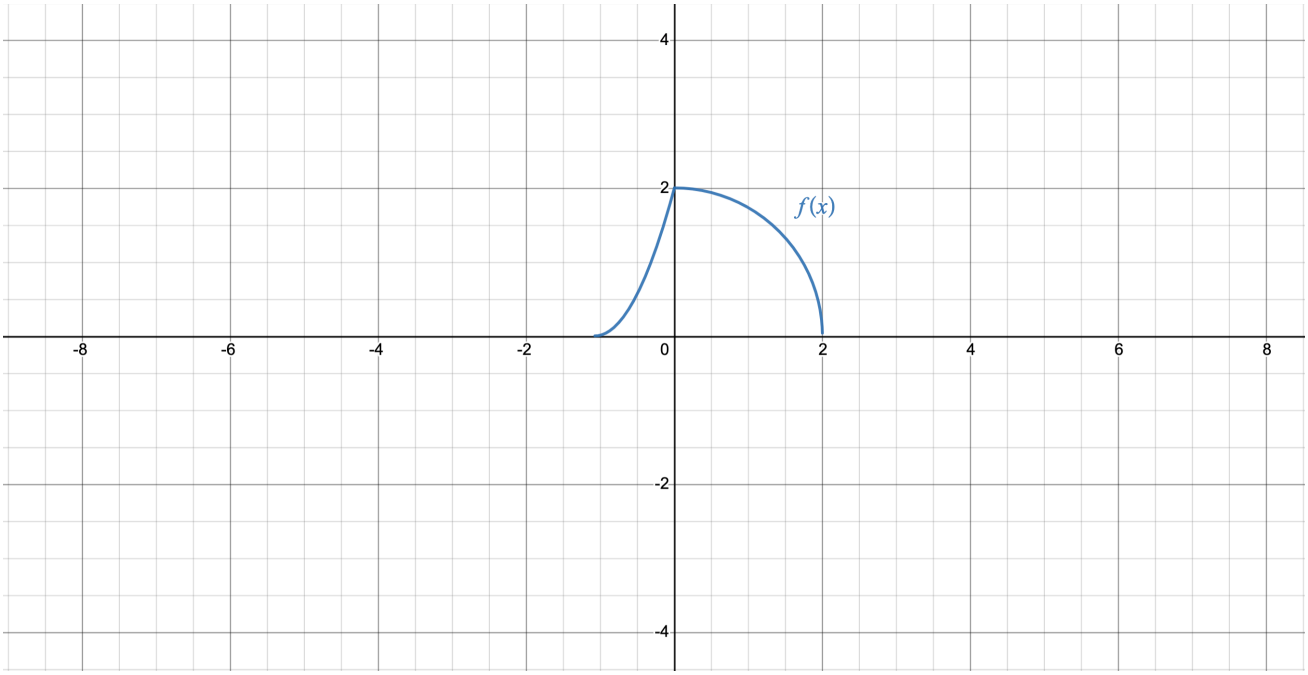
(c) $q(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ \ln x & \text{if } x > 0 \end{cases}$



(d) $k(x) = \tan^{-1}(x)$



6. For the given graph of $f(x)$ in blue; sketch the shape of the graph $-f(x-2)$ on the **same graph**. (4 pts)



7. You are standing 48 meters from the base of a building. You estimate that the angle of elevation to the top of the 82nd floor (the observatory) is 77°. If the total height of the building is another 109 meters above the 82nd floor, what is the height of the building? (5 pts)

8. Given $f(x) = \sqrt{x}$ and $g(x) = x^4 - 2x^2 - 1$ answer the following: (7 pts)

   (a) Find $(g \circ f)(x)$ and find the domain.

   (b) Is $g(x)$ odd, even, or neither? Justify your answer for credit.

9. Find the exact value: (15 pts)

   (a) $\sin\left(\frac{5\pi}{4}\right)$

   (b) $\cos\left(-\frac{\pi}{6}\right)$

   (c) $\arctan(-1)$

   (d) $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

   (e) $\cos\left(\cos^{-1}(1)\right)$

10. For a specific angle θ suppose we know that $\sin\theta > 0$ and θ lies in the interval $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$. What quadrant does θ lie in? (4 pts)

11. Verify the identity: $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta} = \csc\theta$. (5 pts)

12. Suppose we know that $\sec\theta = -\dfrac{9}{4}$ for θ in quadrant II. Find $\sin(2\theta)$. (4 pts)

13. Find all solutions to the following equations: (8 pts)

(a) $\sin(3\theta) = \frac{1}{2}$

(b) $\frac{\sqrt{3}}{2}\tan\theta + \tan\theta\cos(\theta) = 0$

14. Find the exact value for each: (8 pts)

(a) $3\cos^2(-13^\circ) + 3\sin^2(-13^\circ)$

(b) $\cos\left(\frac{\pi}{12}\right)$

15. For $f(x) = 3\cos(2x)$ (9 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $f(x)$. Be sure to label relevant values on x and y axes.