INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted. The exam is worth 100 points. **Assume all questions have answers in the real numbers unless the instructions specify complex numbers.**

Potentially useful formulas:

(i) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ (ii) $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

NAME: ____

1. The following are unrelated: (32 pts)

(a) Simplify: $x^6 - 3x^2 - 1 + (x^2)^4 + (2+2x^3)(x^3+1)$.

Solution:

$$\begin{align} x^6 - 3x^2 - 1 + (x^2)^4 + (2+2x^3)(x^3+1) &= x^6 - 3x^2 - 1 + x^8 + 2x^3 + 2 + 2x^6 + 2x^3 \tag{1}\\ &= x^8 + 3x^6 + 4x^3 - 3x^2 + 1 \tag{2} \end{align}$$

(b) Multiply: $\left(x^{1/2} + x^{3/2}\right)^2$

Solution:

$$\begin{align} \left(x^{1/2} + x^{3/2}\right)^2 &= \left(x^{1/2} + x^{3/2}\right)\left(x^{1/2} + x^{3/2}\right) \tag{3}\\ &= \left(x^{1/2}\right)^2 + x^{1/2}x^{3/2} + x^{1/2}x^{3/2} + \left(x^{3/2}\right)^2 \tag{4}\\ &= x^3 + 2x^2 + x \tag{5} \end{align}$$

(c) Simplify: $\dfrac{a^6}{a^{-3}}\dfrac{(3a)^{-2}}{6}$

Solution:

$$\begin{align} \frac{a^6}{a^{-3}}\frac{(3a)^{-2}}{6} &= \frac{a^6a^3}{1}\frac{1}{6(3a)^2} \tag{6}\\ &= \frac{a^9}{1}\frac{1}{6(9)a^2} \tag{7}\\ &= \frac{a^7}{54} \tag{8} \end{align}$$

(d) Factor completely (If not factorable write NF): $y^3 + 27x^3$

Solution:

$y^3 + 27x^3 = y^3 + (3x)^3 = (y + 3x)\left(y^2 - 3xy + 9x^2\right)$.

(e) Factor completely (If not factorable write NF): $x^3 - 2x^2 + 4x - 8$

Solution:

$$\begin{aligned} x^3 - 2x^2 + 4x - 8 &= x^2(x-2) + 4(x-2) && (9)\\ &= (x-2)(x^2+4) && (10) \end{aligned}$$

(f) Simplify the complex fraction: $\dfrac{\frac{x}{x+2} - \frac{4}{x+2}}{\frac{6}{x+2} - 3}$

Solution:

$$\begin{aligned} \frac{\frac{x}{x+2} - \frac{4}{x+2}}{\frac{6}{x+2} - 3} &= \frac{\frac{x-4}{x+2}}{\frac{6}{x+2} - 3\frac{x+2}{x+2}} && (11)\\ &= \frac{\frac{x-4}{x+2}}{\frac{6-3x-6}{x+2}} && (12)\\ &= \frac{x-4}{x+2} \cdot \frac{x+2}{-3x} && (13)\\ &= -\frac{x-4}{3x} && (14) \end{aligned}$$

(g) Rationalize the denominator: $\dfrac{3 - \sqrt{x}}{3 + \sqrt{x}}$

Solution:

$$\begin{aligned} \frac{3 - \sqrt{x}}{3 + \sqrt{x}} &= \frac{3 - \sqrt{x}}{3 + \sqrt{x}} \cdot \frac{3 - \sqrt{x}}{3 - \sqrt{x}} && (15)\\ &= \frac{9 - 6\sqrt{x} + x}{9 - x} && (16) \end{aligned}$$

(h) Simplify: $(1 - 2i)(1 + 2i) - 3i^4$

Solution:

$$(1-2i)(1+2i)-3i^4 = 1-4i^2-3(i^2)^2 \tag{17}$$
$$= 1-4(-1)-3(-1)^2 \tag{18}$$
$$= 2 \tag{19}$$

(i) Let c be a real number. Find the value of c that makes the factoring of the polynomial true: $2x^2 - cx - 6 = (2x-3)(x+2)$

Solution:

$$2x^2 - cx - 6 = (2x-3)(x+2) \tag{20}$$
$$2x^2 - cx - 6 = 2x^2 + x - 6 \tag{21}$$
$$-cx = x \tag{22}$$
$$-cx - x = 0 \tag{23}$$
$$x(-c-1) = 0 \tag{24}$$

So $-c-1=0$ and solving for c we get $c=-1$ which is the value that results in a true factoring. We could also have found c by observing that $-c=1$ must be true in line (22) so $c=-1$.

2. Simplify: $\dfrac{4x(2x-1)(-2)+3x(2x)^2x}{2x}$ (5 pts)

Solution:

$$\frac{4x(2x-1)(-2)+3x(2x)^2x}{2x} = \frac{-8x(2x-1)+3x^2(2x)^2}{2x} \tag{25}$$
$$= \frac{-16x^2+8x+12x^4}{2x} \tag{26}$$
$$= \frac{2x\left(-8x+4+6x^3\right)}{2x} \tag{27}$$
$$= 6x^3-8x+4 \tag{28}$$

3. Solve each of the following equations: (25 pts)

(a) $5 = x^2 - 4x$

Solution:

$$\begin{align} 5 &= x^2 - 4x \tag{29} \\ x^2 - 4x - 5 &= 0 \tag{30} \\ (x-5)(x+1) &= 0 \tag{31} \end{align}$$

By the multiplicative property of zero we sett $x - 5 = 0$ and $x + 1 = 0$ and find resulting solutions $x = -1, 5$.

(b) $\sqrt{x} - 2 = x - 2$

Solution:

$$\begin{align} \sqrt{x} - 2 &= x - 2 \tag{32} \\ \sqrt{x} &= x \tag{33} \\ x &= x^2 \tag{34} \\ x^2 - x &= 0 \tag{35} \\ x(x-1) &= 0 \tag{36} \end{align}$$

Resulting in possible solutions $x = 0, 1$. Checking these in the original equation, we see they are both solutions.

(c) $\dfrac{x}{x^2-1} + \dfrac{1}{2(x+1)} = \dfrac{x-1}{x^2-1}$

Solution:

$$\begin{align} \frac{x}{x^2-1} + \frac{1}{2(x+1)} &= \frac{x-1}{x^2-1} \tag{37} \\ \frac{x}{(x-1)(x+1)} + \frac{1}{2(x+1)} &= \frac{x-1}{(x-1)(x+1)} \tag{38} \\ 2(x-1)(x+1) \cdot \left(\frac{x}{(x-1)(x+1)} + \frac{1}{2(x+1)} \right) &= 2(x-1)(x+1) \cdot \left(\frac{x-1}{(x-1)(x+1)} \right) \tag{39} \\ 2x + x - 1 &= 2(x-1) \tag{40} \\ x &= -1 \tag{41} \end{align}$$

When we check $x = -1$ in the original equation we see $x = -1$ results in division by zero. So this equation has no solution.

(d) Solve for P: $3-14P=-RP-1$

Solution:

$$3-14P=-RP-1 \tag{42}$$
$$RP-14P=-4 \tag{43}$$
$$P(R-14)=-4 \tag{44}$$
$$P=\frac{-4}{R-14} \tag{45}$$

(e) Solve for r: $I=\dfrac{S}{4\pi r^2}$

Solution:

$$I=\frac{S}{4\pi r^2} \tag{46}$$
$$r^2 I=\frac{S}{4\pi} \tag{47}$$
$$r^2=\frac{S}{4\pi I} \tag{48}$$
$$r=\pm\sqrt{\frac{S}{4\pi I}} \tag{49}$$
$$r=\pm\frac{1}{2}\sqrt{\frac{S}{\pi I}} \tag{50}$$

4. Solve the following inequalities. Justify your answers by using a number line or sign chart. Answers without full justification will not receive full credit. Express all answers in interval notation. (20 pts)

(a) $2+5x\leq -x-1$

Solution:

$$2+5x\leq -x-1 \tag{51}$$
$$6x\leq -3 \tag{52}$$
$$x\leq -\frac{1}{2} \tag{53}$$

Resulting in solution $\left(-\infty,-\frac{1}{2}\right]$.

(b) $x(x-2)^2(x+2) < 0$

Solution:

$$x(x-2)^2(x+2) = 0 \tag{54}$$

Setting up an equation we get $x(x-2)^2(x+2) = 0$ with solutions $x = -2, 0, 2$. Setting up a number line and testing values we get solution: $(-2, 0)$.

(c) $\left|2x + \frac{1}{2}\right| < \frac{1}{2}$

Solution:

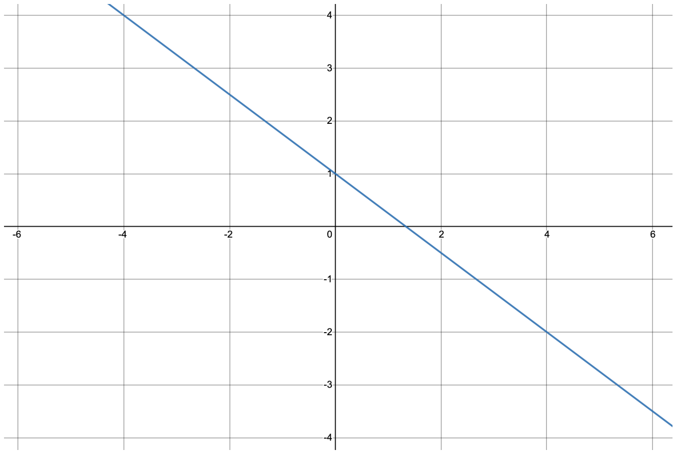
$\left|2x + \frac{1}{2}\right| < \frac{1}{2}$ can be rewritten $-\frac{1}{2} < 2x + \frac{1}{2} < \frac{1}{2}$. Subtracting $\frac{1}{2}$ on each side of the inequalities we get: $-1 < 2x < 0$. Dividing by 2 on all three sides results in $-\frac{1}{2} < x < 0$. In interval notation $\left(-\frac{1}{2}, 0\right)$.

(d) $\frac{x+3}{x} \geq 0$

Solution:

Setting the left hand side equal to zero we get $x = -3$. Setting the denominator equal to zero we get $x = 0$. Putting these values on the real number line and testing values we get the solution: $(-\infty, -3) \cup (0, \infty)$.

5. For the graph of the line below answer the following: (6 pts)



(a) Find the slope of the line.

Solution:

From the graph we see the line crosses through $(0, 1)$ and $(4, -2)$. The slope of the line, m, is found by: $m = \dfrac{-2 - 1}{4 - 0} = -\dfrac{3}{4}$.

(b) Find the equation of the line in $y = mx + b$ (slope-intercept) form.

Solution:

b is the y-intercept of the line which is $(0, 1)$ so $b = 1$. The resulting equation of the line is $y = -\dfrac{3}{4}x + 1$.

6. Find the distance between $(-2, 1)$ and $(0, 7)$. (3 pts)

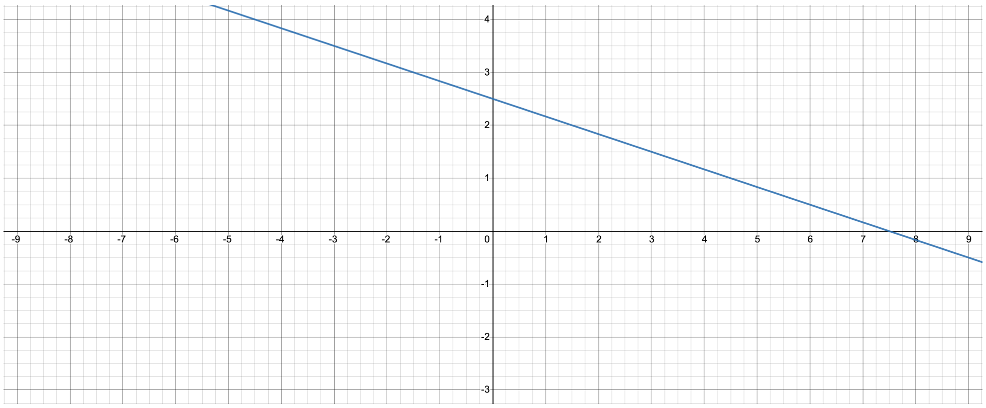
Solution:

Using the distance formula we get:

$$\begin{aligned} \sqrt{(0-(-2))^2 + (7-1)^2} &= \sqrt{4+36} \qquad (55) \\ &= \sqrt{40} \qquad (56) \\ &= 2\sqrt{10} \qquad (57) \end{aligned}$$

7. Graph the line that has slope $m = -\frac{1}{3}$ and crosses through the point $\left(-\frac{3}{2}, 2\right)$. Be sure to label relevant values on the axes. (5 pts)

Solution:



8. Find the value(s) for d such that the midpoint between $(d, -5)$ and $(-1, 3)$ is $\left(\frac{5}{2}, -1\right)$. (5 pts)

Solution:

d is relevant to the midpoint formula for the x-coordinate. The x-coordinate of the midpoint is found by $\frac{d + (-1)}{2} = \frac{5}{2}$. Solving for d:

$$\frac{d + (-1)}{2} = \frac{5}{2} \tag{58}$$

$$d - 1 = 5 \tag{59}$$

$$d = 6 \tag{60}$$