1. Write the word “agree” as your answer to question 1 to indicate that you will abide by the University honor code for this exam. An honor code violation on this exam may result in a zero on the exam or an F in the course.

Solution:

Agree.

2. Sketch the following graphs. Be sure to label any asymptotes and $x$, $y$-intercepts on your axes. (12 pts)

(a) $y = -\left(\frac{2}{3}\right)^x$

Solution:
3. The following are unrelated: (12 pts)

(a) Use the properties of logs to simplify the expression: \( \log_3(54) - \log_3(2) + \log_3(1) + \log_3(9^x) \).

Solution:

\[
\log_3(54) - \log_3(2) + \log_3(1) + \log_3(9^x) = \log_3\left(\frac{54}{2}\right) + 0 + \log_3((3^2)^x) = \log_3(27) + 0 + \log_3(3^{2x}) = 3 + 2x.
\]

(b) Express in terms of sums and differences of logarithms without exponents: \( \ln\left(\frac{e^2}{x^3\sqrt{y}}\right) \)

Solution:

\[
\ln\left(\frac{e^2}{x^3\sqrt{y}}\right) = \ln(e^2) - \ln(x^3\sqrt{y}) = 2\ln(e) - (\ln(x^3) + \ln(\sqrt{y})) = 2 - 3\ln(x) - \frac{1}{2}\ln(y).
\]
4. Solve the following equations. Leave answers in exact form.

(a) \( 3 = \log(2x - 1) \) (6 pts)

**Solution:**

\[
3 = \log(2x - 1) \\
10^3 = 2x - 1 \\
1000 + 1 = 2x \\
x = \frac{1001}{2}
\]

Checking \( x = \frac{1001}{2} \) in the original equation we see that it does solve the equation.

(b) \( 4^{3x+1} = 4^{x-1} \) (6 pts)

**Solution:**

\[
4^{3x+1} = 4^{x-1} \\
3x + 1 = x - 1 \\
2x = -2 \\
x = -1
\]

(c) \( \log_2(x) + \log_2(x + 12) = \log_2(28) \) (6 pts)

**Solution:**

\[
\log_2(x) + \log_2(x + 12) = \log_2(28) \\
\log_2(x(x + 12)) = \log_2(28) \\
x^2 + 12x = 28 \\
x^2 + 12x - 28 = 0 \\
(x + 14)(x - 2) = 0
\]

This results in potential solutions \( x = 2, -14 \). Checking these potential solutions in the original equations results in the only solution of \( x = 2 \).
(d) \(10 = 2e^{4x}\) (6 pts)

Solution:

\[
\begin{align*}
10 &= 2e^{4x} \\
5 &= e^{4x} \\
\ln(5) &= 4x \\
x &= \frac{\ln(5)}{4}
\end{align*}
\]

5. Sketch each angle, \(\theta\), in standard position on the \(x,y\)-axes. Give a separate graph for each.

(a) \(\theta = -\frac{5\pi}{6}\) (5 pts)

Solution:

(b) \(\theta = \pi\) (5 pts)

Solution:
6. Use the image of the triangle to solve for \( x \). (6 pts)

[Diagram of a triangle with a 30° angle]

Solution:

\[
\sin (30^\circ) = \frac{5}{x} \quad (18)
\]
\[
\frac{1}{2} = \frac{5}{x} \quad (19)
\]
\[
\frac{1}{2} = \frac{5}{2x} \quad (20)
\]
\[
2x = 10 \quad (21)
\]
\[
x = 5 \quad (22)
\]

7. Find the following:

(a) \( \sin \left( \frac{7\pi}{4} \right) \) (5 pts)

Solution:

\[
\sin \left( \frac{7\pi}{4} \right) = -\frac{\sqrt{2}}{2}
\]

(b) \( \cos \left( \frac{2\pi}{3} \right) \) (5 pts)

Solution:

\[
\cos \left( \frac{2\pi}{3} \right) = -\frac{1}{2}
\]

(c) \( \tan (-120^\circ) \) (5 pts)

Solution:

\[
\tan (-120^\circ) = \sqrt{3}
\]
(d) \( \sec \left( \frac{\pi}{3} \right) \) (5 pts)

Solution:

\[ \sec \left( \frac{\pi}{3} \right) = 2 \]

8. Evaluate: \( \cos^2 (25.1^\circ) + \sin^2 (25.1^\circ) \) (5 pts)

Solution:

\[ \cos^2 (25.1^\circ) + \sin^2 (25.1^\circ) = 1 \] since \( \cos^2(\theta) + \sin^2(\theta) = 1 \) for all angles.

9. Given \( \tan \theta = \frac{2}{3} \) and \( \theta \) in quadrant III find:

(a) \( \cot \theta \) (5 pts)

Solution:

\[ \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2} \]

(b) \( \cos \theta \) (6 pts)

Solution:

Setting up a reference triangle the hypotenuse is: \( r = \sqrt{2^2 + 3^2} = \sqrt{13} \). So \( \cos \theta = -\frac{3}{\sqrt{13}} \).

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**End of Exam. Formula sheet follows.**

Potentially useful formulas:

(a) Let \( u \) and \( w \) denote positive real numbers, then:
   i. \( \log_b(uv) = \log_b(u) + \log_b(v) \)
   ii. \( \log_b\left(\frac{u}{v}\right) = \log_b(u) - \log_b(v) \)
   iii. \( \log_b(u^c) = c \log_b(u) \) where \( c \) is any real number.
   iv. \( \log_b(u) = \frac{\log_a(u)}{\log_a(b)} \)

(b) \( A = \frac{1}{2} r^2 \theta \)

(c) \( S = r \theta \)

(d) \( \sin^2 \theta + \cos^2 \theta = 1 \)

(e) \( \tan^2 \theta + 1 = \sec^2 \theta \)

(f) \( 1 + \cot^2 \theta = \csc^2 \theta \)