INSTRUCTIONS: Simplify and box all your answers. Write neatly and show all work. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted. The exam is worth 100 points. Assume all questions have answers in the real numbers unless the instructions specify complex numbers.

1. Please write the word “agree” to indicate that you will abide by the University honor code for this exam.

2. The following are unrelated:

   (a) Simplify to rewrite as a polynomial: $z^2 - 2 - (3z^2 - 1)^2$

   Solution:
   
   $$z^2 - 2 - (3z - 1)^2 = z^2 - 2 - (9z^4 - 6z^2 + 1) = -9z^4 + 7z^2 - 3.$$  

   (b) Simplify: $\frac{1}{\sqrt{x}} \left( x^{1/2} - \frac{1}{\sqrt{x}} \right)$

   Solution:
   
   $$\frac{1}{\sqrt{x}} \left( x^{1/2} - \frac{1}{\sqrt{x}} \right) = \frac{x^{1/2}}{\sqrt{x}} - \frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{x} - \frac{1}{x} = 1 - \frac{1}{x} \text{ or } x - \frac{1}{x}.$$  

   (c) Simplify: $\frac{(-3t^3)^2}{r^{5/2} / r^{-1/4}}$

   Solution:
   
   $$\frac{(-3t^3)^2}{r^{5/2} / r^{-1/4}} = 9t^6 r^{5/2 + 1/4} = 9t^6 r^{11/4}.$$  

   (d) Simplify the complex fraction: $\frac{\frac{3}{x - 2} + 1}{\frac{1}{x - 2} - 2}$

   Solution:
   
   $$\frac{\frac{3}{x - 2} + 1}{\frac{1}{x - 2} - 2} = \frac{\frac{3}{x - 2} + \frac{2}{x - 2}}{\frac{1}{x - 2} - \frac{2(x - 2)}{x - 2}} = \frac{\frac{3 + 2}{x - 2}}{\frac{5 - 2x}{x - 2}} = \frac{x + 1}{x - 2} \cdot \frac{x - 2}{5 - 2x} = \frac{1 + x}{5 - 2x}.$$  

   (e) Rewrite in $a + bi$ form: $(2 - 3i)(1 + i)$

   Solution:
   
   $$(2 - 3i)(1 + i) = 2 + 2i - 3i - 3i^2 = 2 - i - 3(-1) = 5 - i.$$
3. Solve each of the following equations:

(a) \(-3(2 - x) = 7x\)

Solution:

\[-3(2 - x) = 7x \quad (1)\]
\[-6 + 3x = 7x \quad (2)\]
\[-6 = 4x \quad (3)\]
\[\frac{3}{2} = x \quad (4)\]

So \(x = \frac{3}{2}\) is the solution to the equation.

(b) \(x^2 - 8x = -16\)

Solution:

\[x^2 - 8x = -16 \quad (5)\]
\[x^2 - 8x + 16 = 0 \quad (6)\]
\[(x - 4)(x - 4) = 0 \quad (7)\]

So the equation is solved when \(x - 4 = 0\) and \(x = 4\) is the solution to the equation.

(c) \(\frac{1}{x - 5} - \frac{5}{x(x - 5)} = 1\)

Solution:

\[\frac{1}{x - 5} - \frac{5}{x(x - 5)} = 1 \quad (8)\]
\[x - 5 = x(x - 5) \quad (9)\]
\[x - 5 = x^2 - 5x \quad (10)\]
\[0 = x^2 - 6x + 5 \quad (11)\]
\[0 = (x - 5)(x - 1) \quad (12)\]

This leads to two potential solutions \(x = 5\) and \(x = 1\), however, checking each solution in the original equation we see that only \(x = 1\) solves the original equation.
(d) Solve for \( p \): \( \frac{p}{q} + p = 2q \)

Solution:

\[
\frac{p}{q} + p = 2q \\
p + qp = 2q^2 \\
p(1 + q) = 2q^2 \\
p = \frac{2q^2}{1 + q}
\]

4. For the given equation of a circle, \( x^2 + y^2 = 1 \), find the equation of the upper half of the circle.

Solution: We start by solving for \( y \).

\[
x^2 + y^2 = 1 \\
y^2 = 1 - x^2 \\
y = \pm \sqrt{1 - x^2}
\]

This results in the equations for the upper and lower half of the circle. The upper half is given by \( y = \sqrt{1 - x^2} \).

5. Solve the following inequalities. Give all final answers in interval notation.

(a) \( x^2 > 9 \)

Solution:

\[
x^2 > 9 \\
x^2 - 9 > 0 \\
(x + 3)(x - 3) > 0
\]

\((x + 3)(x - 3) = 0\) when \( x = -3 \) and \( x = 3 \). Setting up a sign chart or number line we get the solution: \((-\infty, -3) \cup (3, \infty)\).

(b) \( 23 \leq 5x - 3 \)

Solution:

\[
23 \leq 5x - 3 \\
26 \leq 5x \\
\frac{26}{5} \leq x
\]
The answer in interval notation is: \([25, \infty)\).

(c) \(\frac{-2}{x - 3} \geq 0\)

Solution:

Setting up a sign chart or number line we get the solution: \((-\infty, 3)\). Note \(x = 3\) is not included in the answer because this value does not solve the original inequality.

6. (a) Graph the two points \(A(5, 3)\) and \(B(1, -2)\) on the \(xy\) axes. Clearly label values on the axes.

(b) Find the midpoint between the two points.

Solution:

Using the midpoint formula we get \(\left(\frac{5 + 1}{2}, \frac{3 + (-2)}{2}\right) = \left(3, \frac{1}{2}\right)\)

End of exam. Formula sheet on Next Page
Potentially useful formulas:

(i) Equation of a circle: \((x - h)^2 + (y - k)^2 = r^2\)

(ii) \(a^3 - b^3 = (a - b)(a^2 + ab + b^2)\)

(iii) \(a^3 + b^3 = (a + b)(a^2 - ab + b^2)\)

(iv) The quadratic formula: \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\)