INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted. The exam is worth 100 points. **Assume all questions have answers in the real numbers unless the instructions specify complex numbers.**

1. Please write the word "agree" to indicate that you will abide by the University honor code for this exam.

2. The following are unrelated:

   (a) Simplify to rewrite as a polynomial: $z^2 - 2 - (3z^2 - 1)^2$

   Solution:

   $z^2 - 2 - (3z - 1)^2 = z^2 - 2 - (9z^4 - 6z^2 + 1) = -9z^4 + 7z^2 - 3.$

   (b) Simplify: $\dfrac{1}{\sqrt{x}}\left(x^{1/2} - \dfrac{1}{\sqrt{x}}\right)$

   Solution:

   $\dfrac{1}{\sqrt{x}}\left(x^{1/2} - \dfrac{1}{\sqrt{x}}\right) = \dfrac{x^{1/2}}{\sqrt{x}} - \dfrac{1}{(\sqrt{x})^2} = \dfrac{\sqrt{x}}{\sqrt{x}} - \dfrac{1}{x} = 1 - \dfrac{1}{x}$ or $\dfrac{x-1}{x}$.

   (c) Simplify: $\dfrac{\left(-3t^3\right)^2 r^{5/2}}{r^{-1/4}}$

   Solution:

   $\dfrac{\left(-3t^3\right)^2 r^{5/2}}{r^{-1/4}} = 9t^6 r^{5/2+1/4} = 9t^6 r^{11/4}.$

   (d) Simplify the complex fraction: $\dfrac{\frac{3}{x-2} + 1}{\frac{1}{x-2} - 2}$

   Solution:

   $\dfrac{\frac{3}{x-2} + 1}{\frac{1}{x-2} - 2} = \dfrac{\frac{3}{x-2} + \frac{x-2}{x-2}}{\frac{1}{x-2} - 2\frac{x-2}{x-2}} = \dfrac{\frac{3+x-2}{x-2}}{\frac{1-2(x-2)}{x-2}} = \dfrac{\frac{1+x}{x-2}}{\frac{5-2x}{x-2}} = \dfrac{1+x}{x-2} \cdot \dfrac{x-2}{5-2x} = \dfrac{1+x}{5-2x}.$

   (e) Rewrite in $a + bi$ form: $(2 - 3i)(1 + i)$

   Solution:

   $(2 - 3i)(1 + i) = 2 + 2i - 3i - 3i^2 = 2 - i - 3(-1) = 5 - i.$

3. Solve each of the following equations:

(a) $-3(2-x) = 7x$

Solution:

$$-3(2-x) = 7x \tag{1}$$
$$-6 + 3x = 7x \tag{2}$$
$$-6 = 4x \tag{3}$$
$$-\frac{3}{2} = x \tag{4}$$

So $x = -\frac{3}{2}$ is the solution to the equation.

(b) $x^2 - 8x = -16$

Solution:

$$x^2 - 8x = -16 \tag{5}$$
$$x^2 - 8x + 16 = 0 \tag{6}$$
$$(x-4)(x-4) = 0 \tag{7}$$

So the equation is solved when $x - 4 = 0$ and $x = 4$ is the solution to the equation.

(c) $\dfrac{1}{x-5} - \dfrac{5}{x(x-5)} = 1$

Solution:

$$\frac{1}{x-5} - \frac{5}{x(x-5)} = 1 \tag{8}$$
$$x - 5 = x(x-5) \tag{9}$$
$$x - 5 = x^2 - 5x \tag{10}$$
$$0 = x^2 - 6x + 5 \tag{11}$$
$$0 = (x-5)(x-1) \tag{12}$$

This leads to two potential solutions $x = 5$ and $x = 1$, however, checking each solution in the original equation we see that only $x = 1$ solves the original equation.

(d) Solve for p: $\dfrac{p}{q} + p = 2q$

Solution:

$$\frac{p}{q} + p = 2q \tag{13}$$
$$p + qp = 2q^2 \tag{14}$$
$$p(1+q) = 2q^2 \tag{15}$$
$$p = \frac{2q^2}{1+q} \tag{16}$$

4. For the given equation of a circle, $x^2 + y^2 = 1$, find the equation of the upper half of the circle.

Solution: We start by solving for y.

$$x^2 + y^2 = 1 \tag{17}$$
$$y^2 = 1 - x^2 \tag{18}$$
$$y = \pm\sqrt{1 - x^2} \tag{19}$$

This results in the equations for the upper and lower half of the circle. The upper half is given by $y = \sqrt{1 - x^2}$.

5. Solve the following inequalities. Give all final answers in interval notation.

(a) $x^2 > 9$

Solution:

$$x^2 > 9 \tag{20}$$
$$x^2 - 9 > 0 \tag{21}$$
$$(x+3)(x-3) > 0 \tag{22}$$

$(x+3)(x-3) = 0$ when $x = -3$ and $x = 3$. Setting up a sign chart or number line we get the solution: $(-\infty, -3) \cup (3, \infty)$.

(b) $23 \leq 5x - 3$

Solution:

$$23 \leq 5x - 3 \tag{23}$$
$$26 \leq 5x \tag{24}$$
$$\frac{26}{5} \leq x \tag{25}$$

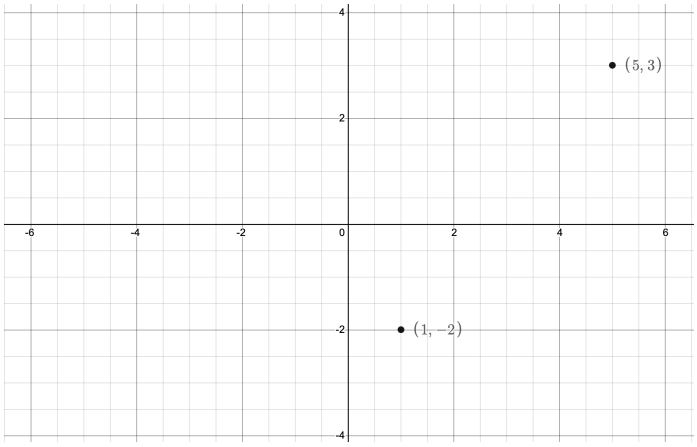
The answer in interval notation is: $[25, \infty)$.

(c) $\dfrac{-2}{x-3} \geq 0$

Solution:

Setting up a sign chart or number line we get the solution: $(-\infty, 3)$. Note $x = 3$ is not included in the answer because this value does not solve the original inequality.

6. (a) Graph the two points $A(5, 3)$ and $B(1, -2)$ on the xy axes. Clearly label values on the axes.



(b) Find the midpoint between the two points.

Solution:

Using the midpoint formula we get $\left(\dfrac{5+1}{2}, \dfrac{3+(-2)}{2}\right) = \left(3, \dfrac{1}{2}\right)$

End of exam. Formula sheet on Next Page

Potentially useful formulas:

(i) Equation of a circle: $(x-h)^2+(y-k)^2=r^2$

(ii) $a^3-b^3=(a-b)(a^2+ab+b^2)$

(iii) $a^3+b^3=(a+b)(a^2-ab+b^2)$

(iv) The quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$