INSTRUCTIONS: Simplify and box all your answers. Write neatly and show all work. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted. The exam is worth 100 points.

Name:_______________________________

1. The following are unrelated. (36 pts)

   (a) Multiply to express as a polynomial: $(2x - 3y)^2$.

      Solution: $(2x - 3y)^2 = (2x - 3y)(2x - 3y) = 4x^2 - 6xy - 6xy + 9y^2 = 4x^2 - 12xy + 9y^2$.

   (b) Evaluate the expression $-\frac{1}{2}x^2 + x^{-1}$ when $x = -\frac{1}{2}$.

      Solution: $\frac{1}{2} \left( -\frac{1}{2} \right)^2 + \left( -\frac{1}{2} \right)^{-1} = -\frac{1}{2} \left( \frac{1}{4} \right) - \frac{1}{2} = \left( -\frac{1}{8} \right) - 2 = -\frac{17}{8}$.

   (c) Simplify: $a^{5/6} \left( a^{-1/6}b^{2/3} \right)^2$

      Solution: $a^{5/6} \left( a^{-1/6}b^{2/3} \right)^2 = a^{5/6} \left( a^{-2/6}b^{4/3} \right) = a^{1/2}b^{4/3}$.

   (d) Multiply to express as a polynomial: $(\sqrt{x-1} + 2)(\sqrt{x-1} - 2)$

      Solution: $(\sqrt{x-1} + 2)(\sqrt{x-1} - 2) = (\sqrt{x-1})^2 - 2\sqrt{x-1} + 2\sqrt{x-1} - 4 = x - 1 - 4 = x - 5$.

   (e) Factor: $x^4 + 2x^2 - 8$ over the real numbers.

      Solution: $x^4 + 2x^2 - 8 = (x^2 - 2)(x^2 + 4) = (x - \sqrt{2})(x + \sqrt{2})(x^2 + 4)$.

   (f) Factor $2x^3 + 16$ over the real numbers.

      Solution: $2x^3 + 16 = 2(x^3 + 8) = 2(x + 2)(x^2 - 2x + 4)$.

   (g) Combine into a single fraction: $\frac{4}{x-2} + \frac{1}{x+3}$

      Solution: $\frac{4}{x-2} + \frac{1}{x+3} = (x + 3) \frac{4}{(x+3)(x-2)} + \frac{1}{(x+3)(x-2)} = \frac{4x + 12}{(x+3)(x-2)} + \frac{x - 2}{(x+3)(x-2)} = \frac{5x + 10}{(x+3)(x-2)}$. 
(h) Simplify (give your answer without negative exponents): \[
\frac{4x^{-2}y^3z^{-1}}{14x^3y^4}
\]
Solution:
\[
\frac{4}{14x^2z} = \frac{2}{7x^2yz}
\]

(i) Simplify the expression: \[
\frac{\frac{1}{x-2}}{1 - \frac{2}{x+1}}
\]
Solution:
\[
\frac{1}{x-2} \cdot \frac{x+1}{x+1} - \frac{2}{x+1} = \frac{1}{x-2} \cdot \frac{x+1}{x+1} = \frac{x+1}{(x-2)(x-1)}.
\]

2. Find all real valued solutions of the following equations: (20 pts)

(a) \[
\frac{24}{3-x} = 4
\]
Solution:
\[
\frac{24}{3-x} = 4
\]
\[
24 = 4(3-x)
\]
\[
24 = 12 - 4x
\]
\[
12 = -4x
\]
\[
-3 = x
\]

(b) \[
5x^3 + 10x^2 - 20x - 40 = 0
\]
Solution:
\[
5x^3 + 10x^2 - 20x - 40 = 0
\]
\[
5(x^3 + 2x^2 - 4x - 8) = 0
\]
\[
x^3 + 2x^2 - 4x - 8 = 0
\]
\[
x^2(x + 2) - 4(x + 2) = 0
\]
\[
(x + 2) (x^2 - 4) = 0
\]
\[
(x + 2)(x - 2)(x + 2) = 0
\]
Setting each factor equal to zero we get: \(x+2 = 0\) and \(x-2 = 0\). Resulting in solutions: \(x = -2, 2\).

(c) \[
q^2 = 3q
\]
Solution:
\[
q^2 = 3q
\]
\[
q^2 - 3q = 0
\]
\[
q(q - 3) = 0
\]
Setting each factor equal to zero we get: \(q = 0\) and \(q - 3 = 0\). Resulting in solutions: \(q = 0, 3\).
(d) \( x + \sqrt{x + 2} = 4 \)

Solution:

\[
\begin{align*}
x + \sqrt{x + 2} &= 4 \quad \text{(15)} \\
\sqrt{x + 2} &= 4 - x \\
x + 2 &= (4 - x)^2 \quad \text{(16)} \\
x + 2 &= 16 - 8x + x^2 \quad \text{(17)} \\
x^2 - 8x + 16 &= x + 2 \quad \text{(18)} \\
x^2 - 9x + 14 &= 0 \quad \text{(19)} \\
(x - 7)(x - 2) &= 0 \quad \text{(20)}
\end{align*}
\]

Setting each factor equal to zero we get: \( x - 7 = 0 \) and \( x - 2 = 0 \). Resulting in potential solutions: \( x = 2, 7 \). Checking each solution we get: \( 2 + \sqrt{2 + 2} = 2 + \sqrt{4} = 2 + 2 = 4 \) and \( 7 + \sqrt{7 + 2} = 7 + \sqrt{9} = 7 + 3 = 10 \). So, the only solution is \( x = 2 \).

3. Solve the following for \( M \): (10 pts)

(a) \( RM = 3M + T \)

Solution:

\[
\begin{align*}
RM &= 3M + T \quad \text{(22)} \\
RM - 3M &= T \quad \text{(23)} \\
M(R - 3) &= T \quad \text{(24)} \\
M &= \frac{T}{R - 3} \quad \text{(25)}
\end{align*}
\]

(b) \( \frac{3}{NM} + \frac{1}{M} = 2 \)

Solution:

\[
\begin{align*}
\frac{3}{NM} + \frac{1}{M} &= 2 \quad \text{(26)} \\
(NM) \left( \frac{3}{NM} + \frac{1}{M} \right) &= (2)(NM) \quad \text{(27)} \\
3 + N &= 2NM \\
3 + N &= 2N \quad \text{(28)} \\
\frac{3 + N}{2N} &= M \quad \text{(29)}
\end{align*}
\]
4. The following are unrelated (10 pts)

(a) Simplify: \((2 - 3i) - (5 - i)\)

Solution: \((2 - 3i) - (5 - i) = 2 - 3i - 5 + i = (2 - 5) + (-3i + i) = -3 - 2i.\)

(b) Simplify: \((5 + 2i)^2\)

Solution: \((5 + 2i)^2 = (5 + 2i)(5 + 2i) = 25 + 10i + 10i + 4i^2 = 25 + 20i + 4(-1) = 25 + 20i - 4 = 21 + 20i.\)

(c) Find all real and complex solutions: \(x^2 + 4x + 5 = 0\)

Using the quadratic formula, we get:
\[
x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{16 - 20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i.
\]

5. Solve the follow inequalities. Give all answers in interval notation: (12 pts)

(a) \(x^2(x - 2) < 0\)

Solution: By solving \(x^2(x - 2) = 0\) we get \(x = 0, 2.\) Setting up a real number line and utilizing test points we get intervals: \((-\infty, 0) \cup (0, 2).\)

(b) \(|x - 2| \leq 4\)

Solution:
\[
|x - 2| \leq 4 \quad (30)
\]
\[
-4 \leq x - 2 \leq 4 \quad (31)
\]
\[
-2 \leq x \leq 6 \quad (32)
\]

With answer: \([-2, 6].\)

(c) \(x^3 + x^2 - 2x \geq 0\)

Solution:
\[
x^3 + x^2 - 2x \geq 0 \quad (33)
\]
\[
x(x^2 + x - 2) \geq 0 \quad (34)
\]
\[
x(x + 2)(x - 1) \geq 0 \quad (35)
\]

Setting \(x(x + 2)(x - 1) = 0\) we get \(x = -2, 0, 1.\) Setting up a real number line and utilizing test points we get intervals: \([-2, 0] \cup [1, \infty).\)
6. For the following points $A(-1, -2)$ and $B(3, 4)$: (12 pts)

(a) Graph and clearly label the two points.

(b) Find the distance between $A$ and $B$.

Solution: 
\[ d = \sqrt{(3 - (-1))^2 + (4 - (-2))^2} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}. \]

(c) Find the midpoint between $A$ and $B$.

Solution: 
\[ \left( \frac{3 - 1}{2}, \frac{4 - 2}{2} \right) = \left( \frac{2}{2}, \frac{2}{2} \right) = (1, 1). \]

(d) Find the point on segment $AB$ that is three-fourths of the way from $A$ to $B$.

Solution: This point can be found by taking the midpoint between $(1,1)$ and $(3,4)$: 
\[ \left( \frac{3 + 1}{2}, \frac{4 + 1}{2} \right) = \left( \frac{2}{2}, \frac{5}{2} \right). \]

Potentially useful equations:

(i) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(ii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$