1. Simplify the following: (11 pts)

(a) \( \frac{a^7b^2c^{-1}}{ab^5c^{-2}} \)

Solution: \( \frac{a^7b^2c^{-1}}{ab^5c^{-2}} = \frac{a^7c}{b^3c} = \frac{a^7c}{b^3} \)

(b) \( \sqrt[3]{27x^8y^3z^4} \)

Solution: \( \sqrt[3]{27x^8y^3z^4} = 3x^2yz \sqrt[3]{x^2z} \)

2. (12 pts)

(a) Express as a polynomial: \((x + 4)(x + 3) - (2x - 3)(x - 5)\)

Solution: \((x + 4)(x + 3) - (2x - 3)(x - 5) = x^2 + 7x + 12 - (2x^2 - 13x + 15) = -x^2 + 20x - 3\)

(b) Simplify: \(\frac{2x + 3}{x - 1}\)

Solution: \(\frac{2x + 3}{x - 1} \cdot \frac{x^2}{x^2} = \frac{2x^3 + 3}{x - x^2}\)

3. Solve the following equations: (21 pts)

(a) \(x(3x + 2) = 2\)

Solution: \(x(3x + 2) = 2 \Rightarrow 3x^2 + 2x - 2 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 - 4(3)(-2)}}{2(3)} = \frac{-2 \pm \sqrt{28}}{6} = \frac{-2 \pm \sqrt{7}}{3}\)

(b) \(\sqrt{7x + 2} + x = 6\)

Solution: \(\sqrt{7x + 2} + x = 6 \Rightarrow \sqrt{7x + 2} = 6 - x \Rightarrow 7x + 2 = (6 - x)^2 \Rightarrow 7x + 2 = 36 - 12x + x^2 \Rightarrow 0 = x^2 - 19x + 34 \Rightarrow (x - 17)(x - 2) = 0 \Rightarrow x = 2, 17\). Note: \(x = 17\) is not a solution, the only answer is \(x = 2\).
(c) \(3 = 5 + \ln(x + 2)\)

Solution: \(3 = 5 + \ln(x + 2) \Rightarrow -2 = \ln(x + 2) \Rightarrow e^{-2} = x + 2 \Rightarrow x = e^{-2} - 2\)

4. Find the domain of \(\frac{x^2 - 1}{\sqrt{1 - x}}\) (6 pts)

Solution: \(1 - x > 0 \Rightarrow 1 > x\). Answer: \((-\infty, 1)\)

5. Sketch a graph of each function and state the domain and range for each function: (14 pts)

(a) \(y = \ln(x - 1)\)

Solution: From the graph we get: Domain: \((1, \infty)\) Range: \((-\infty, \infty)\)

(b) \(y = |x| + 2\)

Solution: From the graph we get: Domain: \((-\infty, \infty)\) Range: \([2, \infty)\)
6. Find the quotient and remainder if \( f(x) \) is divided by \( p(x) \). \( f(x) = 3x^4 - 4x^3 + x^2 + x + 5 \) and \( p(x) = x^3 - 2x + 7 \) (6 pts)

Solution: By long division, the quotient is \( 3x - 4 \) and the remainder is \( 7x^2 - 28x + 33 \)

7. Expand, without exponents, using properties of logs: \( \log_7 \left( \frac{x^3 \sqrt{y}}{z^2} \right) \) (6 pts)

Solution: \( \log_7 \left( \frac{x^3 \sqrt{y}}{z^2} \right) = \log_7 (x^3 \sqrt{y}) - \log_7 (z^2) = 3 \log_7 x + \frac{1}{2} \log_7 y - 2 \log_7 z \)

8. One thousand trout, each 1 year old, are introduced into a large pond. It is predicted that the number \( N(t) \) still alive after \( t \) years will be given by the equation \( N(t) = 1000(0.9)^t \). At what time will 500 trout be alive? (6 pts)

Solution: \( 500 = 1000(0.9)^t \Rightarrow \frac{1}{2} = (0.9)^t \Rightarrow t = \log_{0.9} \left( \frac{1}{2} \right) \)

9. Find the exact value: (20 pts)

(a) \( \sin \left( \frac{5\pi}{6} \right) \)

Solution: \( \sin \left( \frac{5\pi}{6} \right) = \frac{1}{2} \)

(b) \( \cot \left( -\frac{3\pi}{4} \right) \)

Solution: \( \cot \left( -\frac{3\pi}{4} \right) = 1 \)

(c) \( \tan^{-1} \left( -\sqrt{3} \right) \)

Solution: \( \tan^{-1} \left( -\sqrt{3} \right) = -\frac{\pi}{3} \)

(d) \( \arcsin \left( \sin \left( \frac{5\pi}{4} \right) \right) \)

Solution: \( \arcsin \left( \sin \left( \frac{5\pi}{4} \right) \right) = \arcsin \left( \sin \left( -\frac{\pi}{4} \right) \right) = -\frac{\pi}{4} \)

(e) \( \cos \left( \cos^{-1} \left( 2 \right) \right) \)

Solution: \( \cos \left( \cos^{-1} \left( 2 \right) \right) \) DNE since \( \cos^{-1}(2) \) is not defined.
10. Verify the identity: \( \cos \theta + \sin \theta \tan \theta = \sec \theta \) (6 pts)

One possible solution:

\[
\cos \theta + \sin \theta \tan \theta = \cos \theta + \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta
\]

11. Find all solutions to the following equations on \([0, 2\pi]\): (12 pts)

(a) \( \tan^2 \theta = \tan \theta \)

Solution: \( \tan^2 \theta - \tan \theta = 0 \Rightarrow \tan \theta (\tan \theta - 1) = 0 \) and \( \theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4} \)

(b) \( 2 \cos(3\theta) = 1 \)

Solution: \( \cos(3\theta) = \frac{1}{2} \Rightarrow 3\theta = \frac{\pi}{3} + 2n\pi \) and \( 3\theta = -\frac{\pi}{3} + 2n\pi \) where \( n \) is any integer. \( \theta = \frac{\pi}{9} + \frac{2n\pi}{3} \)

On \([0, 2\pi]\), \( \theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9} \)

12. Find the exact value: (12 pts)

(a) \( \cos \left( \frac{\pi}{8} \right) \)

Solution: \( \cos \left( \frac{\pi}{8} \right) = \cos \left( \frac{\pi}{2} - \frac{\pi}{4} \right) = \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2} \)

(b) \( \sin(70^\circ) \cos(10^\circ) - \cos(70^\circ) \sin(10^\circ) \)

Solution: \( \sin(70^\circ) \cos(10^\circ) - \cos(70^\circ) \sin(10^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2} \)

13. For \( y = 3 \sin \left( 2 \left( x - \frac{\pi}{4} \right) \right) \) (18 pts)

(a) Identify the amplitude

Solution: 3

(b) Identify the period.

Solution: \( \frac{2\pi}{2} = \pi \)
(c) Identify the phase shift

Solution: Right by $\frac{\pi}{4}$

(d) Sketch two cycles of the curve and make sure to label values on the axes.

Solution:
Formulas that may be useful:

\[ \sin(a + b) = \sin a \cos b + \cos a \sin b \]

\[ \sin(a - b) = \sin a \cos b - \cos a \sin b \]

\[ \cos(a + b) = \cos a \cos b - \sin a \sin b \]

\[ \cos(a - b) = \cos a \cos b + \sin a \sin b \]

\[ \sin(2\theta) = 2 \sin \theta \cos \theta \]

\[ \cos(2\theta) = \cos^2 \theta - \sin^2 \theta \]

\[ \cos(2\theta) = 1 - 2 \sin^2 \theta \]

\[ \cos(2\theta) = 2 \cos^2 \theta - 1 \]

\[ \sin \left( \frac{\theta}{2} \right) = \pm \sqrt{\frac{1 - \cos \theta}{2}} \]

\[ \cos \left( \frac{\theta}{2} \right) = \pm \sqrt{\frac{1 + \cos \theta}{2}} \]

Equation of a circle: \((x - h)^2 + (y - k)^2 = r^2\)

Linear speed \(L = r\omega\), where \(r\) is the radius and \(\omega\) is the angular speed

Arc length: \(s = r\theta\) where \(r\) is the radius and \(\theta\) is in radians

Area of a sector: \(A = \frac{1}{2}r^2\theta\) where \(r\) is the radius and \(\theta\) is in radians