INSTRUCTIONS: Books, notes, and electronic devices are not permitted. The exam is worth 100 points. Box your answers. Write neatly and show all work. A correct answer with incorrect or no supporting work may receive no credit.

Name:______________________________

1. Sketch the graph of: (10 pts)
\[
\begin{cases}
\sqrt{x+1} & x \leq 1 \\
x^3 & 1 < x \leq 2 \\
-x + 4 & x > 2
\end{cases}
\]

2. For \( f(x) = \sqrt{x-1} \) and \( g(x) = x^2 + 1 \) determine the following functions and their respective domains: (18 pts, 6 pts each)
   
   (a) \((fg)(x)\)
   
   (b) \(\left(\frac{g}{f}\right)(x)\)

   (c) \((g \circ f)(x)\)
3. Consider the polynomial function \( y = x^3 + 2x^2 - 4x - 8 \). Sketch the curve (make sure to label the zeros and y-intercept) (10 pts)

4. Consider \( x^3 - x^2 - 10x - 8 = 0 \) (10 pts)
   
   (a) Use the rational roots theorem to list all possible rational solutions of the equation.

   (b) Find all solutions of the equation.
5. Determine whether the following functions are odd, even, or neither. (10 pts)

(a) \( w(x) = \frac{|x|}{-3} \)

(b) \( f(x) = x^2 - 2x - 1 \)

6. Find a third degree polynomial with integer coefficients that has zeros \( x = 2i, 7 \). (6 pts)

7. Consider \( f(x) = x^2 - 6x - 5 \) (10 pts)

(a) Rewrite the quadratic function in standard form and identify the coordinates of the vertex.

(b) Does this function have a maximum or minimum value? Give a brief explanation why.
8. Consider the following rational function: \( r(x) = \frac{x^4 - x^3 + 8x^2 - 8x}{-x^3 + x^2 - 2x} \) (18 pts, 6 pts each)

(a) Find the coordinates of any holes in the rational expression \( r(x) \). If there are none write NONE.

(b) Find any horizontal or oblique (slant) asymptotes of \( r(x) \). If there are none write NONE.

(c) Find any vertical asymptotes of \( r(x) \). If there are none write NONE.
9. A farmer wishes to put a fence around a rectangular field. The farmer will then divide the field into three equal sized rectangular plots by placing two fences parallel to one of the sides (see picture). The farmer can afford 100 ft of fencing. The farmer wishes to find the dimensions of the rectangular field that will maximize the total enclosed area. If the field has width \( w \) and length \( l \), set up (but do not solve) an equation to represent the area of the field as a function of either length or width. (8 pts)