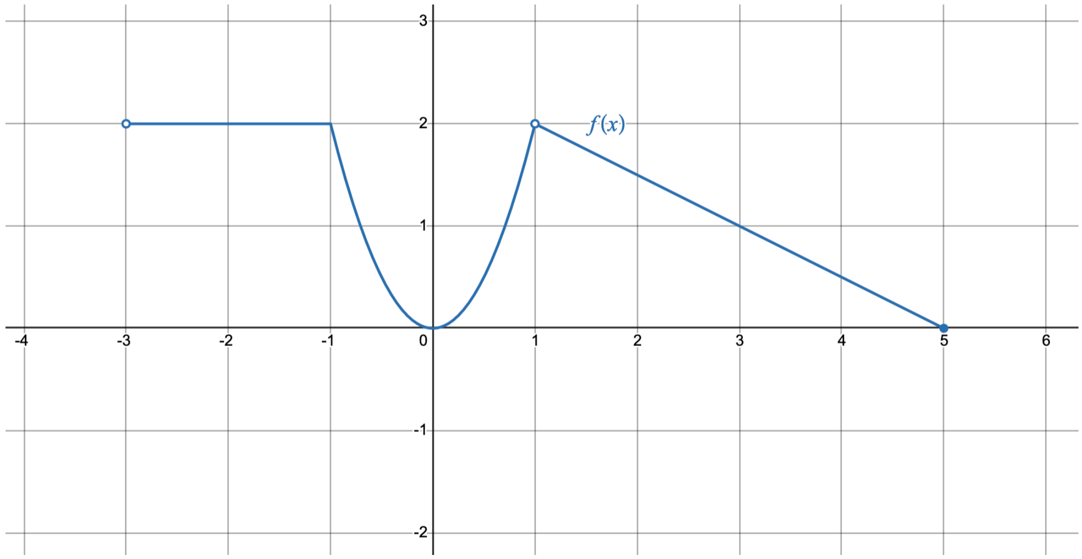
1. Answer the following for the given graph of a function $y = f(x)$. Give answers in interval notation where relevant (10 pts):



(a) Identify the domain of f.

Solution:

$\boxed{(-3, 1) \cup (1, 5]}$

(b) Identify the the range of f.

Solution:

$\boxed{[0, 2]}$

(c) Find the midpoint between $(-2, f(-2))$ and $(3, f(3))$.

Solution:

$$x_m = \frac{3 + (-2)}{2} = \frac{1}{2} \tag{1}$$

$$y_m = \frac{f(3) + f(-2)}{2} = \frac{1 + 2}{2} = \frac{3}{2} \tag{2}$$

The midpoint is $\boxed{\left(\frac{1}{2}, \frac{3}{2}\right)}$.

(d) Solve $f(x) = 0$. If there are no solutions write "NO SOLUTIONS."

Solution:

$f(x) = 0$ at $\boxed{x = 0 \text{ and } x = 5}$.

(e) Find $(f \circ f)(5)$. If the value does not exist write "DNE."

Solution:

$$
\begin{aligned}
(f \circ f)(5) &= f(f(5)) && (3)\\
&= f(0) && (4)\\
&= \boxed{0} && (5)
\end{aligned}
$$

(f) Find the x-values where $f(x) < 2$. Give your answer in interval notation.

Solution:

$\boxed{(-1, 1) \cup (1, 5]}$

(g) Identify a restriction of the domain so that f is one-to-one and has the same range as in part (b).

Solution:

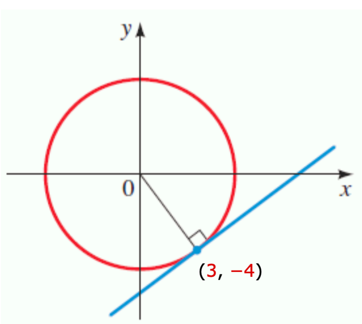
One possible choice is: $\boxed{[-1, 0]}$.

(h) Use your domain restriction to calculate $f^{-1}(2)$.

Solution:

From the graph, restricted to the domain in part (g), we know that $f(-1) = 2$. Therefore $\boxed{f^{-1}(2) = -1}$.

2. Consider the image and find the equation of the tangent line in blue that passes through $(3, -4)$. (6 pts)



Solution:

In order to find the equation of a line, we need the slope and a point on the line. We already have a point on the line: $(3, -4)$. We therefore must find the slope.

From the diagram, we know that the blue tangent line forms a right angle with the black line from $(0, 0)$ to $(3, -4)$ (along the radius of the circle). We can find the slope of the black radial line because we know two points on it:

$$
\begin{aligned}
m_{radial} &= \frac{-4 - 0}{3 - 0}\\
&= -\frac{4}{3}
\end{aligned}
$$

Since the blue tangent line is perpendicular (or ”normal”) to the black line, it must have the negative reciprocal slope:

$$m_{tangent} = -\frac{1}{-\frac{4}{3}} = \frac{3}{4}$$

Combining this information in the point-slope form of a line, we obtain the equation of the blue tangent line:

$$\begin{aligned} y - (-4) &= \frac{3}{4}(x-3) \\ y + 4 &= \frac{3}{4}x - \frac{9}{4} \\ \Aboxed{y &= \frac{3}{4}x - \frac{25}{4}} \end{aligned}$$

3. Find the domain of the following functions. Express your answers in interval notation. (12 pts)

   (a) $g(x) = \dfrac{x}{x^2 + 12x}$

   Solution:

   We must exclude all x-values that make the denominator zero.

   $$\begin{aligned} 0 &= x^2 + 12x \\ &= x(x+12) \end{aligned}$$

   therefore the domain requires that $x \neq 0, -12$. The domain is $\boxed{(-\infty, -12) \cup (-1, 0) \cup (0, \infty)}$.

   (b) $s(r) = |-4r^3 + 12|$

   Solution:

   There are no even roots and no quotients. The domain is $\boxed{(-\infty, \infty)}$.

   (c) $f(x) = \dfrac{1}{(x+2)\sqrt{2-3x}}$

   Solution:

   First, we must find and exclude all x-values that make the denominator equal to zero.

   $$(x+2)\sqrt{2-3x} = 0$$

   $x + 2 = 0$ when $x = -2$, while $\sqrt{2-3x} = 0$ when $2 - 3x = 0$, or $x = 2/3$. Therefore the domain must exclude $x = -2$ and $x = 2/3$. So far, we have $(-\infty, -2) \cup (-2, 2/3) \cup (2/3, \infty)$.

   Next, we cannot have a negative number in the square root. Therefore $2 - 3x \geq 0$, or $x \leq 2/3$. This restriction alone gives $(-\infty, 2/3]$.

   The domain is the overlap between these two, or x-values that satisfy both requirements. Therefore the domain is $\boxed{(-\infty, -2) \cup \left(-2, \frac{2}{3}\right)}$.

4. The cost of producing a certain television can be modeled by: $C(t) = 20t + 300$ where C is the cost to produce a television in US dollars and t is time in years after present day (with present day occuring at $t = 0$). Answer the following: (6 pts)

   (a) What are the units of the slope of the model?

   Solution:

   $\boxed{\text{US dollars per year}}$ for the slope of 20.

   (b) Is the cost increasing, decreasing, or staying the same each year?

   Solution:

   The slope is positive, so the cost is $\boxed{\text{increasing}}$.

   (c) What is the C-intercept of the model?

   Solution:

   $\boxed{C = \$300}$

   (d) Briefly explain, in words, what the C-intercept represents.

   Solution:

   $\boxed{\text{Currently (at } t = 0\text{), the cost to produce a television is 300 dollars.}}$

5. Fluid is leaking out of a container. How much fluid remains in the container is governed by Torricelli's Law: $V(t) = 50\left(1 - \frac{t}{20}\right)^2$ where t is in minutes and V is measured in ft^3. (6 pts)

   (a) Find the net change in volume of fluid from 0 minutes to 10 minutes.

   Solution:

$$
\begin{aligned}
V(10) - V(0) &= 50\left(1 - \frac{10}{20}\right)^2 - 50\left(1 - \frac{0}{20}\right)^2 && (6)\\
&= 50\left(\frac{1}{2}\right)^2 - 50 && (7)\\
&= \frac{50}{4} - 50 && (8)\\
&= -\frac{75}{2} && (9)
\end{aligned}
$$

   Hence the net change in volume is $\boxed{-\frac{75}{2}}$ ft^3.

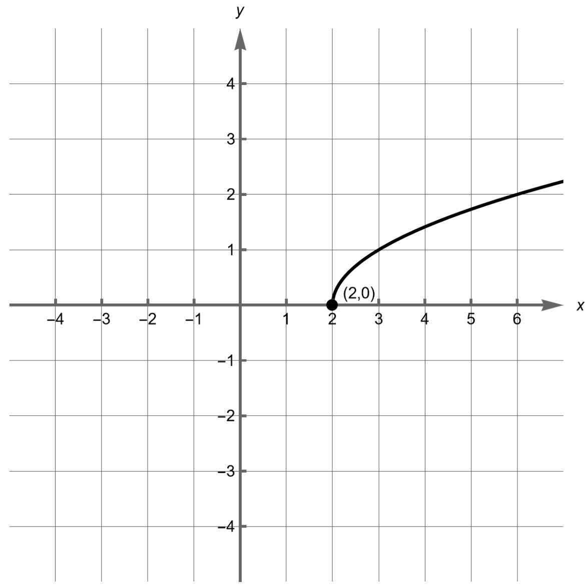
   (b) Briefly explain, in words, what the net change found in part (a) means for the fluid in the container.

   Solution:

   $\boxed{\text{The volume of the fluid in the container decreased by } \frac{75}{2} \text{ ft}^3 \text{ (or 37.5 ft}^3\text{) in the first 10 minutes.}}$

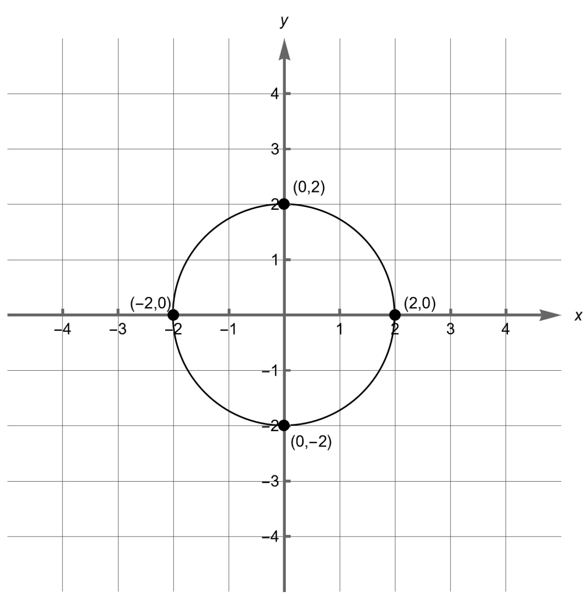
6. Sketch the shape of the graph of each of the following on the provided axes. Make sure to label relevant value(s) on your axe(s) (10 pts)

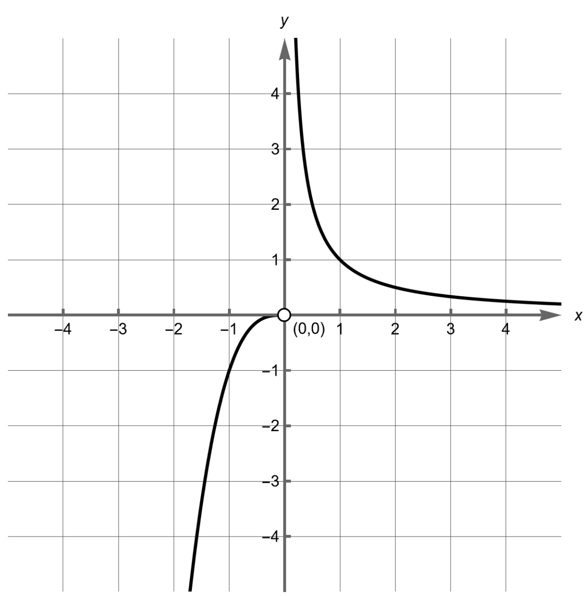
(a) $f(x) = \sqrt{x-2}$



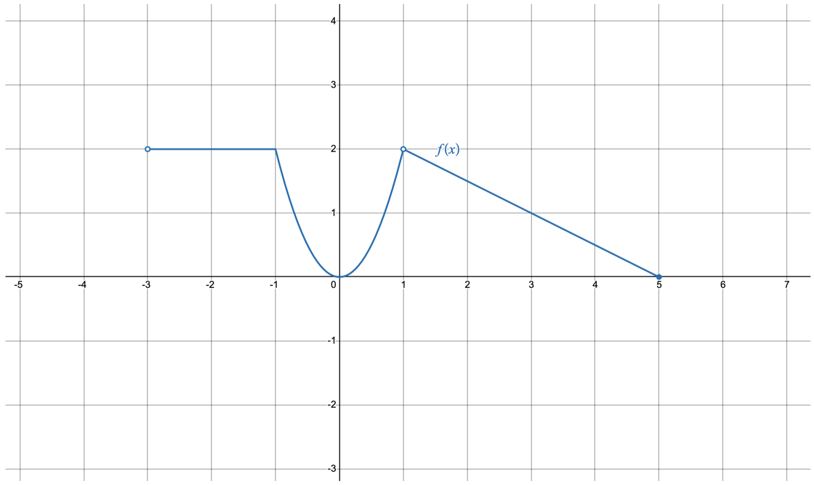
(b) $x^2 + y^2 = 4$

(c) $r(x) = \begin{cases} x^3 & \text{if} \quad x < 0 \\ \frac{1}{x} & \text{if} \quad x \geq 0 \end{cases}$

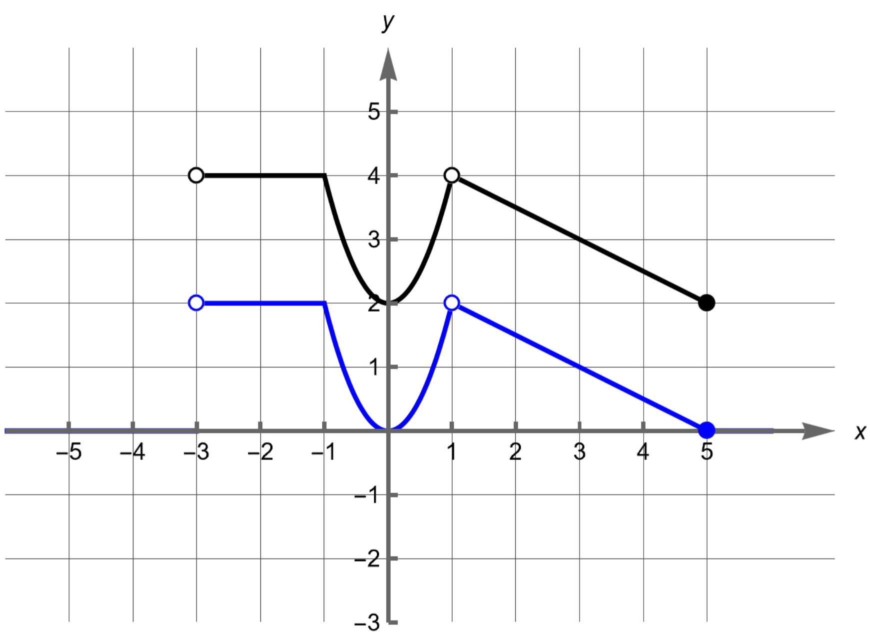




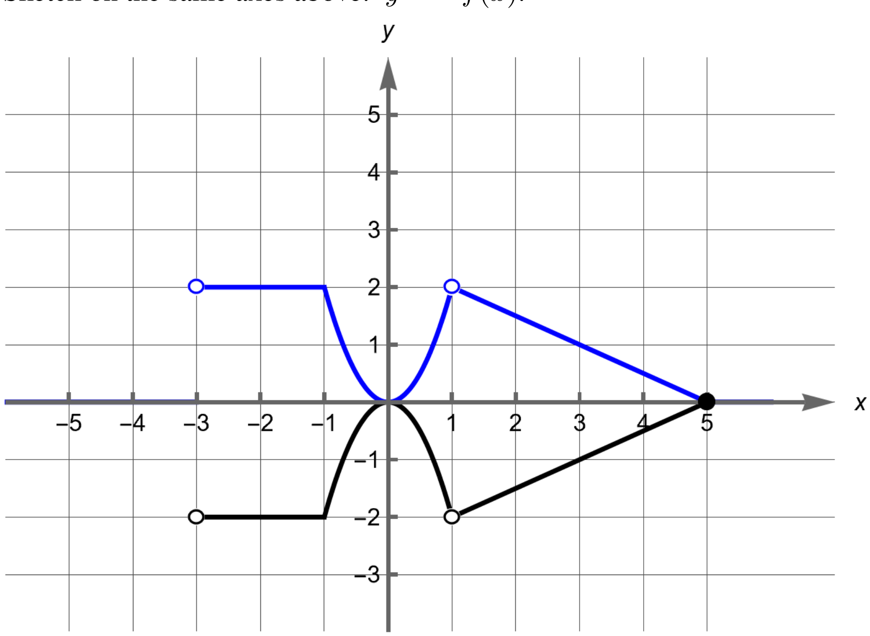
7. For the graph below, this is the same graph as question 1, answer the following (6 pts):



(a) Sketch on the same axes above: $y = f(x) + 2$.



(b) Sketch on the same axes above: $y = -f(x)$.



8. For $g(x) = \dfrac{1}{2x} + 1$ compute the following for real number constant a and nonzero constant h: (7 pts)

(a) $g(a)$

Solution:

$$\boxed{\frac{1}{2a} + 1}$$

(b) $g(a+h)$

Solution:

$$\boxed{\frac{1}{2(a+h)} + 1}$$

(c) $\dfrac{g(a+h) - g(a)}{h}$

Solution:

$$\frac{g(a+h) - g(a)}{h} = \frac{\frac{1}{2(a+h)} + 1 - \left(\frac{1}{2a} + 1\right)}{h} \tag{10}$$

$$= \frac{\frac{1}{2(a+h)} - \frac{1}{2a}}{h} \tag{11}$$

$$= \frac{a - (a+h)}{2a(a+h)h} \tag{12}$$

$$= \frac{-h}{2a(a+h)h} \tag{13}$$

$$= \boxed{-\frac{1}{2a(a+h)}} \tag{14}$$

9. For $P(x) = 2x^3 + 6x^2 + 4x$ answer the following. (15 pts)

(a) i. Identify the term that dominates the end behavior of $P(x)$:

Solution:

The term with the highest power dominates the end behavior of a polynomial. For $P(x)$, this term is $\boxed{2x^3}$.

ii. Based on your answer to part (a) fill in the blanks for $P(x)$:

$y \to$ ____ as $x \to -\infty$ and $y \to$ ____ as $x \to \infty$.

Solution:

$y \to \boxed{-\infty}$ as $x \to -\infty$ and $y \to \boxed{\infty}$ as $x \to \infty$, since 3 is an odd power and the coefficient, 2, is positive.

(b) Find the y-intercept of $P(x)$.

Solution: The y-intercept is the value of P when $x = 0$: $P(0) = 2 \cdot 0^3 + 6 \cdot 0^2 + 4 \cdot 0 = \boxed{0}$

(c) i. Find all zeros of $P(x)$.

Solution:

The zeros of $P(x)$ are all the x valuess that make $P(x) = 0$. To find them we first factor $P(x)$:

$$\begin{aligned} P(x) &= 2x^3 + 6x^2 + 4x && (15) \\ &= 2x(x^2 + 3x + 2) && (16) \\ &= 2x(x+1)(x+2) && (17) \end{aligned}$$

By the multiplicative property of 0, we set factor each equal to 0 and solve for x to get our zeros:

$$\begin{aligned} x &= 0 && (18) \\ x &= -1 && (19) \\ x &= -2 && (20) \end{aligned}$$

So $\boxed{x = 0, x = -1, x = -2}$

ii. Identify the multiplicity of each zero.

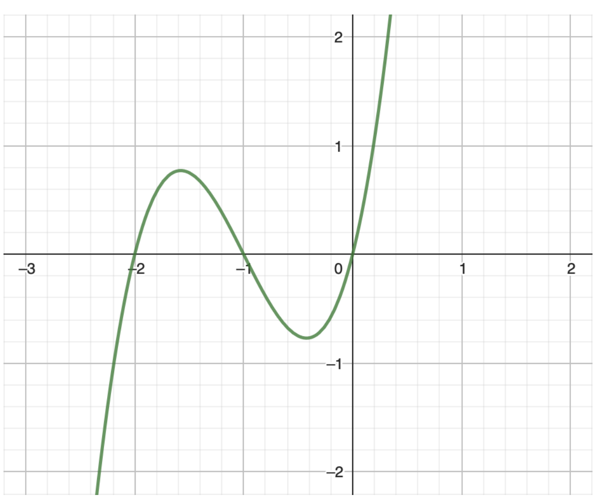
Solution:

Each factor appears once in the factored form, so they all have multiplicity $\boxed{1}$

(d) Sketch the graph of $P(x)$ using the above information. Be sure to label all intercept(s).

Solution:

Our function's graph crosses the graph at all the x-intercepts since they all have odd multiplicity. Using the end behavior and the y-intercept, our plot should look like this:



10. For $h(x) = \frac{1}{2}\sqrt{x} - 1$ and $r(x) = \sqrt{2}\sqrt{x+1}$, answer the following: (10 pts)

(a) Find $n(x) = (r \circ h)(x)$.

Solution:

$$\begin{aligned}
(r \circ h)(x) &= r(h(x)) && (21)\\
&= r\left(\frac{1}{2}\sqrt{x} - 1\right) && (22)\\
&= \sqrt{2}\sqrt{\frac{1}{2}\sqrt{x} - 1 + 1} && (23)\\
&= \sqrt{2}\sqrt{\frac{1}{2}\sqrt{x}} && (24)\\
&= \sqrt{2}\frac{\sqrt[4]{x}}{\sqrt{2}} && (25)\\
&= \boxed{\sqrt[4]{x}} && (26)
\end{aligned}$$

(b) Find the domain of $n(x)$.

Solution: The domain of $h(x)$ is $[0, \infty)$ and the domain of $n(x)$ is also $[0, \infty)$, so the combined function's domain is $\boxed{[0, \infty)}$.

(c) Find $r^{-1}(x)$ (you may assume that $r(x)$ is one-to-one).

Solution:

We set $y = r(x)$, solve for y, then swap x and y to get our inverse:

$$\begin{aligned}
y &= \sqrt{2}\sqrt{x+1} && (27)\\
y^2 &= 2(x+1) && (28)\\
\frac{1}{2}y^2 &= x + 1 && (29)\\
\frac{1}{2}y^2 - 1 &= x && (30)\\
\boxed{\frac{1}{2}x^2 - 1} &= y && (31)
\end{aligned}$$

So the inverse function is: $\boxed{r^{-1}(x) = \frac{1}{2}x^2 - 1}$ on the restricted domain $[0, \infty)$.

11. The following are unrelated. (6 pts)

(a) Is $f(x) = x^3 - 2x + 4$ odd, even, or neither? Fully justify your answer to earn credit.

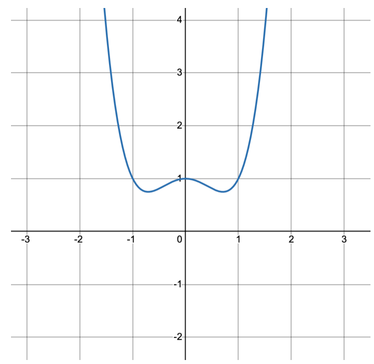
Solution:

We calculate $f(-x)$:

$$f(-x) = (-x)^3 - 2(-x) + 4 \quad (32)$$
$$= -x^3 + 2x + 4 \quad (33)$$

This doesn't equal either $f(x)$ or $-f(x)$, so the function is $\boxed{neither}$ even nor odd.

(b) Is the graph given that of an odd function, even function, or neither?



Solution:

As the graph is symmetric about the y-axis, the function is $\boxed{even}$.

12. A farmer is preparing to grow strawberries in a field. The number of strawberries produced by each plant depends on how densely the plants are grown. If only one strawberry plant were grown in the field, it would be expected to produce 300 strawberries. For each additional plant added to the field, the number of strawberries produced by each plant would be 3 fewer. Let n represent the number of plants grown in the field. (6 pts)

(a) Find a function that represents the total number of strawberries produced in the field, T, as a function of n.

Solution:

The total number of strawberries produced will be the number of strawberries produced per plant times the number of plant.

The number of additional plants after the first is $n - 1$, so the number of strawberries produced per plant is $300 - 3(n - 1)$. We multiply this by the number of plants, n, to get the total number of strawberries:

$$T(n) = (300 - 3(n - 1))n \quad (34)$$
$$= (300 - 3n + 3)n \quad (35)$$
$$= (303 - 3n)n \quad (36)$$
$$= \boxed{303n - 3n^2} \quad (37)$$

(b) Use your answer from part (a) to determine how many plants should be grown in the field to maximize the number of strawberries produced.

Solution:

$T(n)$ is a quadratic, so we can find the value of n where it has a maximum by finding the vertex. Comparing $T(n)$ to the expanded form:

$$T(n) = -3n^2 + 303n \tag{38}$$
$$= an^2 + bn + c \tag{39}$$

we see that $a = -3$ and $b = 303$ and $c = 0$, so the maximum is at

$$h = -\frac{b}{2a} \tag{40}$$
$$= -\frac{303}{2(-3)} \tag{41}$$
$$= \frac{303}{6} \tag{42}$$
$$= 50.5 \tag{43}$$

Since $T(50) = T(51)$ then planting either $\boxed{50}$ or $\boxed{51}$ strawberry plants maximizes the number of strawberries produced.