1. The following are unrelated: (8 pts)

   (a) In the following comma-separated list, identify and write down all integers. In your answer: **Include all expressions that simplify to an integer.**

   $$\left\{4,\ -\frac{1}{2},\ \pi,\ \frac{0}{5},\ 3\sqrt[4]{16},\ 5(3^{-1}),\ \frac{\pi}{2},\ 2^0\right\}$$

   Solution: $\boxed{\left\{4,\ \frac{0}{5},\ 3\sqrt[4]{16},\ 2^0\right\}}$ since $\frac{0}{5} = 0,\ 3\sqrt[4]{16} = 3 \cdot 2 = 6,\ \text{and}\ \ 2^0 = 1$

   (b) Express the quantity without using absolute value:

   i. $|-x+4|$ where $x > 4$

   Solution: When $x > 4$, $-x + 4 < 0$, therefor $|-x+4| = -(-x+4) = \boxed{x-4}$

   ii. $|\sqrt{2} - 1|$

   Solution: Since $\sqrt{2} > 1$, $|\sqrt{2} - 1| = \boxed{\sqrt{2} - 1}$

2. The following are unrelated: (13 pts)

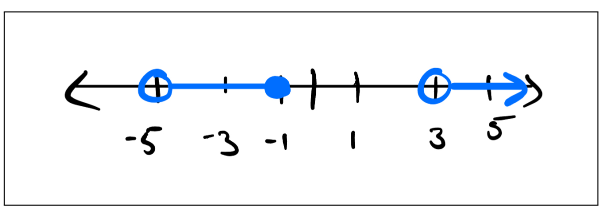
   (a) Divide/subtract as indicated: $\frac{1}{4} - \frac{2}{\frac{12}{3}}$

   Solution:

   $$\frac{1}{4} - \frac{2}{\frac{12}{3}} = \frac{1}{4} - 2 \cdot \frac{3}{12} \tag{1}$$
   $$= \frac{1}{4} - 2 \cdot \frac{1}{4} \tag{2}$$
   $$= \frac{1}{4} - \frac{2}{4} \tag{3}$$
   $$= \boxed{-\frac{1}{4}} \tag{4}$$

   (b) Graph $(-5, 1] \cup (3, \infty)$ on the real number line.

   Solution:

   (c) Express the statement "t is not less than 9" as an inequality.

   Solution: $\boxed{t \geq 9}$

   (d) If $x > 0$ and $y < 0$ is the expression $-x^3y^4$ positive, negative, or zero?

   Solution: Since $x^3 > 0$ and $y^4 > 0$, then $-x^3y^4 < 0$ and $-x^3y^4$ is $\boxed{Negative}$.

3. Simplify (4 pts): $\dfrac{|5|-|3+4|}{1-|-3+1|}$

Solution:

$$\begin{aligned}\frac{|5|-|3+4|}{1-|-3+1|} &= \frac{|5|-|7|}{1-|-2|} && (5)\\ &= \frac{5-7}{1-2} && (6)\\ &= \frac{-2}{-1} && (7)\\ &= \boxed{2} && (8)\end{aligned}$$

4. The following are unrelated: (16 pts)

(a) Evaluate: $\sqrt{15}\sqrt{6}$

Solution:

$$\begin{aligned}\sqrt{15}\sqrt{6} &= \sqrt{(3)(5)}\sqrt{(3)(2)} && (9)\\ &= \sqrt{(3^2)(5)(2)} && (10)\\ &= \boxed{3\sqrt{10}} && (11)\end{aligned}$$

(b) Simplify and combine: $\sqrt{50x^4y}-\sqrt{18x^4y}$

Solution:

$$\begin{aligned}\sqrt{50x^4y}-\sqrt{18x^4y} &= \sqrt{(2)(25)x^4y}-\sqrt{(2)(9)x^4y} && (12)\\ &= \sqrt{(2)(5^2)x^4y}-\sqrt{(2)(3^2)x^4y} && (13)\\ &= 5x^2\sqrt{2y}-3x^2\sqrt{2y} && (14)\\ &= \boxed{2x^2\sqrt{2y}} && (15)\end{aligned}$$

(c) Simplify the expression: $\sqrt{9x^2+9}$

Solution:

$$\begin{aligned}\sqrt{9x^2+9} &= \sqrt{9\left(x^2+1\right)} && (16)\\ &= \sqrt{9}\sqrt{(x^2+1)} && (17)\\ &= \boxed{3\sqrt{x^2+1}} && (18)\end{aligned}$$

(d) Evaluate: $12(-x)^3$ for $x=-\dfrac{1}{2}$

Solution:

$$\begin{aligned}12(-x)^3 &= 12\left(-\left(-\frac{1}{2}\right)\right)^3 && (19)\\ &= 12\left(\frac{1^3}{2^3}\right) && (20)\\ &= 12\left(\frac{1}{8}\right) && (21)\\ &= \frac{12}{8} && (22)\\ &= \boxed{\frac{3}{2}} && (23)\end{aligned}$$

5. The following are unrelated: (12 pts)

(a) Simplify (Give your answer without negative exponents): $\dfrac{a^{10}b^{-2}a^{-1}}{b^{11}} - \left(2a^{1/9}a^{-1/3}\right)^3$

Solution:

$$\frac{a^{10}b^{-2}a^{-1}}{b^{11}} - \left(2a^{1/9}a^{-1/3}\right)^3 = a^9b^{-13} - \left(2a^{-2/9}\right)^3 \tag{24}$$
$$= a^9b^{-13} - 8a^{-\frac{2}{3}} \tag{25}$$
$$= \boxed{\frac{a^9}{b^{13}} - \frac{8}{a^{\frac{2}{3}}}} \tag{26}$$

(b) Multiply and simplify: $\left(\sqrt{x-2}-3\right)^2$

Solution:

$$\left(\sqrt{x-2}-3\right)^2 = \left(\sqrt{x-2}-3\right)\left(\sqrt{x-2}-3\right) \tag{27}$$
$$= \left(\sqrt{x-2}\right)^2 - 2\cdot 3\cdot\sqrt{x-2} + 3^2 \tag{28}$$
$$= x - 2 + 9 - 6\sqrt{x-2} \tag{29}$$
$$= \boxed{x + 7 - 6\sqrt{x-2}} \tag{30}$$

(c) Multiply: $\dfrac{1}{4\sqrt{x}}\left(2x^{3/2} + 12\sqrt{x}\right)$

Solution:

$$\frac{1}{4\sqrt{x}}\left(2x^{3/2} + 12\sqrt{x}\right) = \frac{1}{4\sqrt{x}}\cdot 2x^{3/2} + 3 \tag{31}$$
$$= \frac{x^{-1/2}}{2}x^{3/2} + 3 \tag{32}$$
$$= \boxed{\frac{x}{2} + 3} \tag{33}$$

6. The following are unrelated: (12 pts)

(a) Simplify: $\dfrac{2x(3x)^2x - 3x(x+2)(-2)}{2x}$

Solution:

$$\begin{aligned}
\frac{2x(3x)^2x - 3x(x+2)(-2)}{2x} &= \frac{2x9x^2x + 6x(x+2)}{2x} && (34)\\
&= \frac{18x^4 + 6x^2 + 12x}{2x} && (35)\\
&= \frac{2x\left(9x^3 + 3x + 6\right)}{2x} && (36)\\
&= \boxed{9x^3 + 3x + 6} && (37)
\end{aligned}$$

(b) Perform the multiplication and simplify: $\dfrac{z^3 - 8}{2z^2 + 12z + 18} \cdot \dfrac{z^2 + 3z}{3z - 6}$

Solution:

$$\begin{aligned}
\frac{z^3 - 8}{2z^2 + 12z + 18} \cdot \frac{z^2 + 3z}{3z - 6} &= \frac{(z-2)(z^2 + 2z + 4)}{2(z^2 + 6z + 9)} \cdot \frac{z(z+3)}{3(z-2)} && (38)\\
&= \frac{(z-2)(z^2 + 2z + 4)}{2(z+3)(z+3)} \cdot \frac{z(z+3)}{3(z-2)} && (39)\\
&= \boxed{\frac{z(z^2 + 2z + 4)}{6(z+3)}} && (40)
\end{aligned}$$

(c) Simplify the compound fraction: $\dfrac{\frac{3}{x-1} - \frac{1}{x}}{x - 1}$

Solution:

$$\begin{aligned}
\frac{\frac{3}{x-1} - \frac{1}{x}}{x - 1} &= \frac{\frac{3x}{x(x-1)} - \frac{x-1}{x(x-1)}}{x - 1} && (41)\\
&= \frac{\frac{2x+1}{x(x-1)}}{\frac{x-1}{1}} && (42)\\
&= \frac{2x + 1}{x(x-1)} \cdot \frac{1}{x - 1} && (43)\\
&= \boxed{\frac{2x + 1}{x(x-1)^2}} && (44)
\end{aligned}$$

7. Determine whether or not $x = 4$ is a solution of the following equation (show all work to receive credit): $\frac{1}{x} - \frac{1}{x-6} = \frac{5}{4}$ (3 pts)

Solution:

Plugging $x = 4$ to the left hand side, we obtain:

$$\begin{aligned} \frac{1}{4} - \frac{1}{4-6} &= \frac{1}{4} - \frac{1}{(-2)} && (45) \\ &= \frac{1}{4} + \frac{1}{2} && (46) \\ &= \frac{1}{4} + \frac{2}{4} && (47) \\ &= \frac{3}{4} && (48) \end{aligned}$$

Since the resulting value is not $\frac{5}{4}$ then we conclude that $x = 4$ is $\boxed{\text{NOT a solution}}$ to the given equation.

8. Solve the following equation over the complex numbers: $3z^2 + 9 = 0$. (4 pts)

Solution:

$$\begin{aligned} 3z^2 + 9 &= 0 && (49) \\ 3z^2 &= -9 && (50) \\ z^2 &= -3 && (51) \\ z &= \boxed{\pm\sqrt{3}i} && (52) \end{aligned}$$

9. An explosion causes a rock to be flung upward at an initial velocity of 64 ft/s at time $t = 0$. An engineer wants to estimate at what times, t, the rock will reach a height of 48 ft. The engineer decides that solving the equation $-16t^2 + 64t = 48$ will give the answer. Use the equation to solve for t. (4 pts)

Solution:

This equation is quadratic. We rearrange to get a zero on one side:

$$16t^2 - 64t + 48 = 0.$$

To make this equation easier to solve, we can divide everything by 16. We then factor:

$$\begin{aligned} t^2 - 4t + 3 &= 0 \\ (t-3)(t-1) &= 0 \end{aligned}$$

The solutions are $\boxed{t = 1 \text{ s}}$ and $\boxed{t = 3 \text{ s}}$.

10. Solve the equation (4 pts): $\frac{2}{3}x - 2 = 1 - \frac{1}{2}x$

Solution:

$$\begin{aligned}
\frac{2}{3}x - 2 &= 1 - \frac{1}{2}x && (53)\\
\frac{2}{3}x + \frac{1}{2}x &= 2 + 1 && (54)\\
\frac{4}{6}x + \frac{3}{6}x &= 3 && (55)\\
\frac{7}{6}x &= 3 && (56)\\
x &= \frac{6}{7}(3) && (57)\\
x &= \boxed{\frac{18}{7}} && (58)
\end{aligned}$$

11. Solve each of the following equations: (8 pts)

(a) Solve for p: $T = \frac{2wp - 4p}{r}$

(b) $\frac{-2y+6}{y^3} + \frac{1}{y^2} = \frac{1}{y}$

Solution:

(a)

$$\begin{aligned}
T &= \frac{2wp - 4p}{r} && (59)\\
Tr &= 2wp - 4p && (60)\\
Tr &= p(2w - 4) && (61)\\
\frac{Tr}{2w - 4} &= p && (62)\\
p &= \frac{Tr}{2w - 4} && (63)
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{-2y+6}{y^3} + \frac{1}{y^2} &= \frac{1}{y} && (64)\\
y^3\left(\frac{-2y+6}{y^3} + \frac{1}{y^2}\right) &= \left(\frac{1}{y}\right)y^3 && (65)\\
y^3\left(\frac{-2y+6}{y^3}\right) + y^3\left(\frac{1}{y^2}\right) &= \left(\frac{1}{y}\right)y^3 && (66)\\
-2y + 6 + y &= y^2 && (67)\\
6 - y &= y^2 && (68)\\
0 &= y^2 + y - 6 && (69)\\
0 &= (y+3)(y-2) && (70)
\end{aligned}$$

Thus $\boxed{y = -3}$ and $\boxed{y = 2}$. Note that both of these do not result in division by zero in the original equation and are in fact solutions.

12. Solve the following inequalities. Justify your answers by using a number line or sign chart if needed. Answers without full justification will not receive full credit. Express all answers in interval notation. (12 pts)

(a) $9 - 3x \geq -5x$

Solution:

$$\begin{aligned} &9 - 3x \geq -5x \\ &9 + 2x \geq 0 \\ &2x \geq -9 \\ &x \geq -\frac{9}{2} \end{aligned}$$

The solution is $\boxed{\left[-\frac{9}{2}, \infty\right)}$.

(b) $x^4(x-2)(x+3) \geq 0$

Solution:
First find the values of x such that $x^4(x-2)(x+3) = 0$ These are $x = 0, -3, 2$. We can therefore split up the number line into the intervals

$$(-\infty, -3) \cup (-3, 0) \cup (0, 2) \cup (2, \infty)$$

and test points on each interval:

$$\begin{aligned} x = -4 &\implies x^4(x-2)(x+3) \implies (-4)^4(-6)(-1) > 0 \\ x = -1 &\implies x^4(x-2)(x+3) \implies (-1)^4(-3)(2) < 0 \\ x = 1 &\implies x^4(x-2)(x+3) \implies 1^4(-1)(4) < 0 \\ x = 3 &\implies x^4(x-2)(x+3) \implies 3^4(1)(6) > 0 \end{aligned}$$

Therefore $x^4(x-2)(x+3) > 0$ on the intervals $(-\infty, -3)$ and $(2, \infty)$. Since we can have $x^4(x-2)(x+3)^2 = 0$ as well and this occurs at $x = -3, 0, 2$ (see the $\geq$ in the original inequality) the solution is $\boxed{(-\infty, -3] \cup [0, 0] \cup [2, \infty)}$.

(c) $|4x - 3| < 5$

Solution:
$|4x - 3| < 5$ is solved when $-5 < 4x - 3 < 5$. Adding 3 to all three pieces we get $-2 < 4x < 8$ and then dividing by 4 we get: $-\frac{1}{2} < x < 2$ so the solution is $\boxed{\left(-\frac{1}{2}, 2\right)}$.

Alternative solution:

First determine the values of x such that $|4x - 3| = 5$. Removing the absolute value gives $4x - 3 = \pm 5$, with solutions $x = -\frac{1}{2}, 2$. We can then split up the real line into the intervals

$$\left(\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 2\right) \cup (2, \infty)$$

and test points in each interval:

$$\begin{aligned} x = -1 &\implies |4x - 3| \implies |-7| > 5 \\ x = 0 &\implies |4x - 3| \implies |-3| < 5 \\ x = 3 &\implies |4x - 3| \implies |9| > 5 \end{aligned}$$

Therefore $|4x - 3| < 5$ on the interval $\boxed{\left(-\frac{1}{2}, 2\right)}$, and this is the solution.