
INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources), help from another person, are not permitted during the exam. **Give all answers in exact form.**

Potentially useful formulas:

$$1. a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$10. a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$2. \text{ Circle: } (x - h)^2 + (y - k)^2 = r^2$$

$$3. \text{ Arc length: } s = r\theta$$

$$11. \text{ Area of a sector: } A = \frac{1}{2}r^2\theta$$

$$4. \sin(a - b) = \sin a \cos b - \sin b \cos a$$

$$12. \sin(a + b) = \sin a \cos b + \sin b \cos a$$

$$5. \cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$13. \cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$6. \cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$14. \sin(2\theta) = 2 \sin \theta \cos \theta$$

$$7. \cos(2\theta) = 2 \cos^2 \theta - 1$$

$$15. \cos(2\theta) = 1 - 2 \sin^2 \theta$$

$$8. \sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$16. \cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$9. \sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$17. \cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

NOTE: YOU MAY TEAR OFF THIS PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

- i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.
- ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE
- iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE.
- iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND MOVE TO THE EXAM SUBMISSION AREA.

Name: _____

1. Add/subtract/divide/multiply as indicated: (6 pts)

(a) $\frac{7}{3} - \frac{\frac{3}{2}}{9} + 3^{-1}$

(b) $(e^x)^3 - e^x (e^{2x} - 2e^{-x})$

2. The following are unrelated. (10 pts)

(a) Place the correct symbol, $<$, $>$, or, $=$ in the space between each of the following pair of numbers.

i. $\frac{5}{12}$ $\frac{4}{9}$

ii. $-\sqrt{2}$ $-\sqrt{3}$

(b) Let x and y be real numbers such that $x < 0$ and $y < 0$. Determine whether the following expression is positive, negative, or zero: $-\frac{x^7}{8y^3}$.

(c) Rewrite the following without absolute value symbols:

i. $|e - 1|$

ii. $|x - 3|$ where $x < 3$

3. The following are unrelated. Leave answers without negative exponents. (6 pts)

(a) Simplify: $(2r - 3)^2 - \frac{54(3r^2)^{-3}}{r^{-7}}$

(b) Evaluate and simplify: $\frac{\cos(\frac{\pi}{3})}{\sqrt{2}} - 8^{-1/2}$

4. The following are unrelated. (6 pts)

(a) Simplify: $\frac{x^2 + 1}{x^2 - 1} \cdot \frac{x^2 - 3x + 2}{3x^4 + 3x^2}$

(b) Multiply and simplify: $(\sqrt{x} - 3\sqrt{y})(\sqrt{x} + 3\sqrt{y})$

5. The following are unrelated. (6 pts)

(a) Simplify: $\frac{1 + \frac{1}{x-2}}{\frac{x}{x-2} - \frac{1}{x-2}}$

(b) Simplify the following: $\log(1) + \frac{\log_2(\frac{1}{2})}{\log_2(8)} - e^{\ln 4}$ (Your answer should have no logarithms)

6. The following are unrelated: (7 pts)

(a) Multiply and simplify: $(2 - 3i)(-1 + i)$

(b) Solve the following equation over the complex numbers. $x^2 + 4x + 5 = 0$

7. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (12 pts)

(a) Solve for a : $\frac{5}{6}a - \frac{1}{2} = \frac{a}{2} - \frac{2}{3}$

(b) Solve for x : $\sqrt{x^2 + 4} + 2 = 2x$

(c) Solve for t : $\ln(t) = \ln(2) + \ln(t + 2)$

8. For the two points $P(5, -1)$ and $Q(2, 3)$: (6 pts)

(a) Find the slope of the line through the two points.

(b) Find the equation of the line that passes through the points P and Q .

9. Consider the functions: $f(x) = \frac{1}{x-1}$ and $g(x) = e^x$. (8 pts)

(a) Find $\left(\frac{g}{f}\right)(x)$.

(b) Find $(f \circ g)(x)$.

(c) Find the domain of $(f \circ g)(x)$. Give your answer in interval notation.

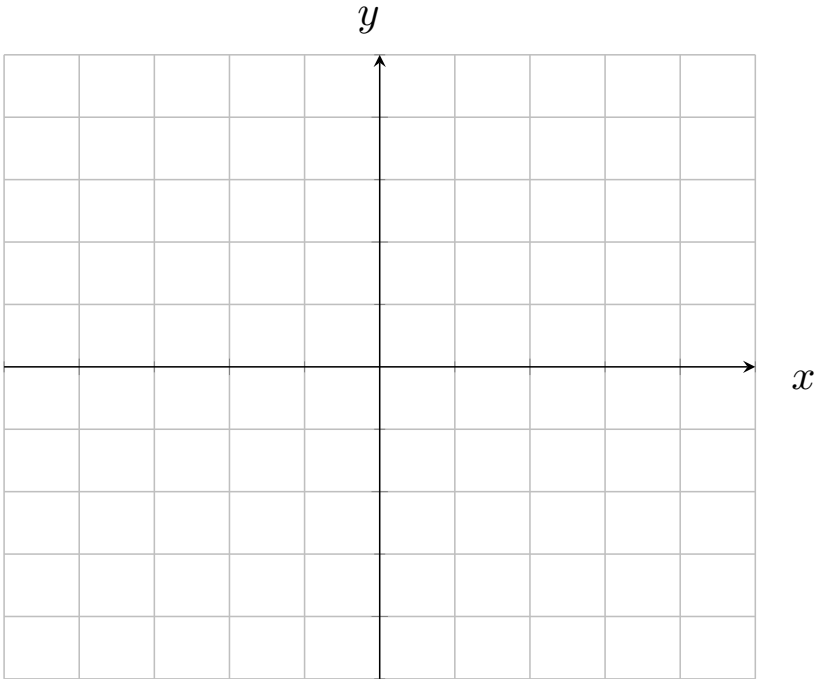
10. For the polynomial function $P(x) = 2(x - 1)^2(x + 2)$ (10 pts)

(a) Indicate on a graph or use arrow notation to indicate the end behavior of $P(x)$.

(b) Find all zeros of $P(x)$ and identify the multiplicity for each.

(c) Find the y -intercept.

(d) Sketch the graph of $P(x)$ using parts (a) through (c).



11. For the rational function $R(x) = \frac{x^3 + 7x^2 + 12x}{x^2 - 3x}$ answer the following (10 pts):

(a) Find the x -coordinate of any hole(s). If there are no hole(s) write NONE.

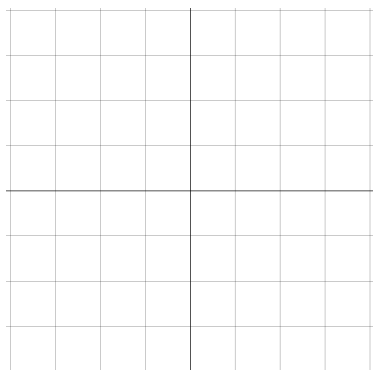
(b) Find the y -coordinate of any hole(s). If there are no hole(s) write NONE.

(c) Determine the end behavior of $R(x)$.

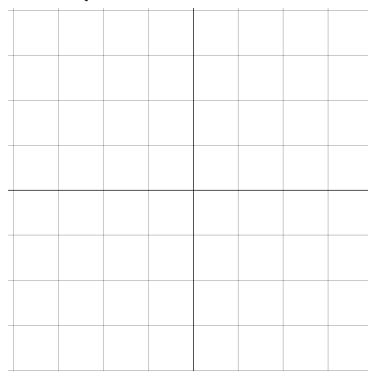
(d) Find all vertical asymptote(s). If there are none write NONE.

12. Sketch the graph of the following functions. **Label all intercepts and asymptotes as appropriate.** (12 pts)

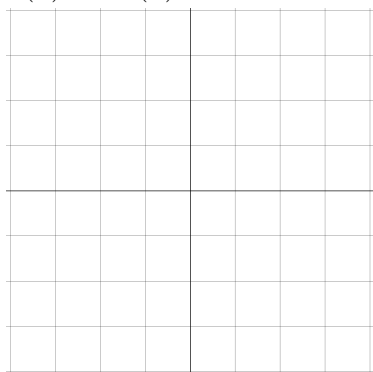
(a) $f(x) = x^2 - 2$



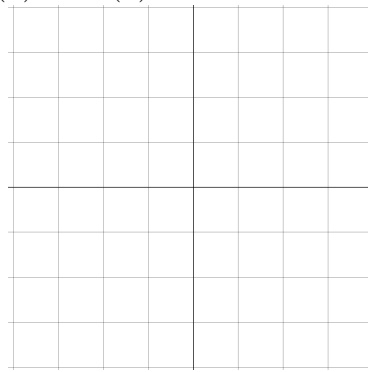
(c) $q(x) = \begin{cases} 1 & \text{if } x \leq -2 \\ 2x & \text{if } -2 < x \leq 0 \\ \ln(x) & \text{if } x > 0 \end{cases}$



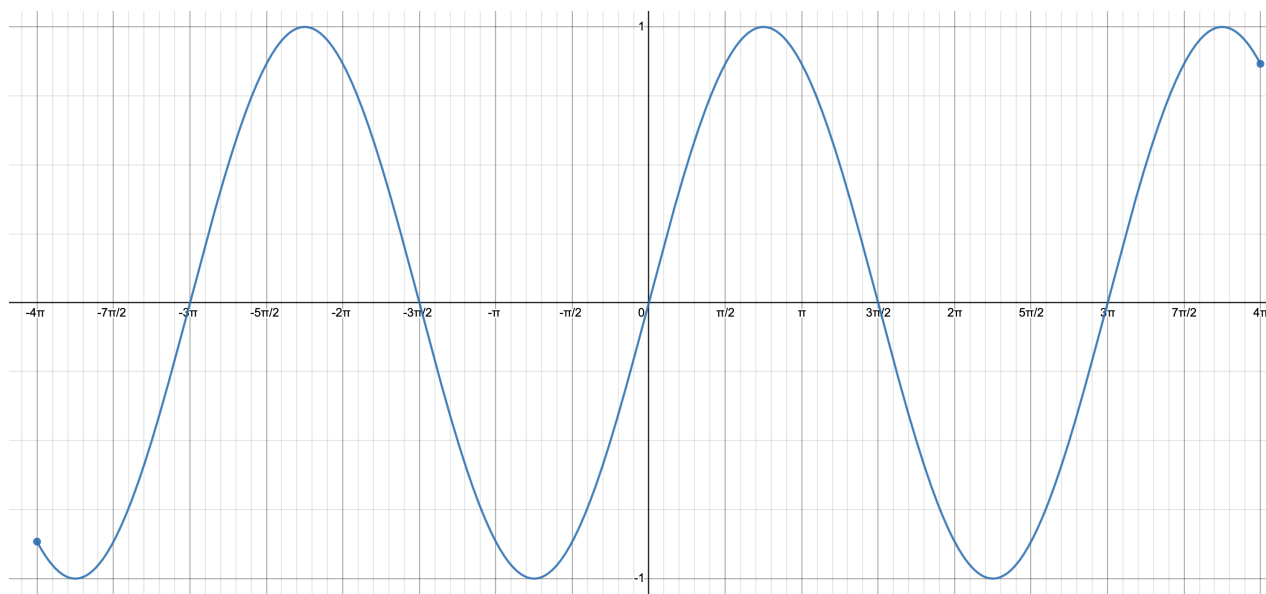
(b) $h(x) = \sin(x)$ on the restricted domain $[0, 2\pi]$



(d) $k(x) = \csc(x)$ on the restricted domain $[0, 2\pi]$



13. Use the graph of the function $T(x)$, with domain $[-4\pi, 4\pi]$, below to answer the following: (8 pts)



(a) Solve the equation $T(x) = 0$.

(b) Solve the inequality $T(x) > 0$. Give your answer in interval notation.

(c) Identify a restriction of the domain of $T(x)$ such that the range is preserved and the graph is one-to-one.

(d) Is $T(x)$ odd, even, or neither?

14. Find the exact value: (15 pts)

(a) $\sin(-\pi)$

(d) $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

(b) $\cos\left(\frac{5\pi}{6}\right)$

(e) $\sin(75^\circ)$

(c) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

(f) $\cos\left(\frac{\pi}{8}\right)$

15. Verify the identity: $\frac{\csc \theta - \sin \theta}{\sin \theta} = \cot^2 \theta$. (4 pts)

16. Find all solutions to the following equations: (8 pts)

(a) $\cos \theta \tan \theta - 3 \tan \theta = 0$

(b) $\sin \left(\frac{\theta}{3} \right) = \frac{1}{2}$

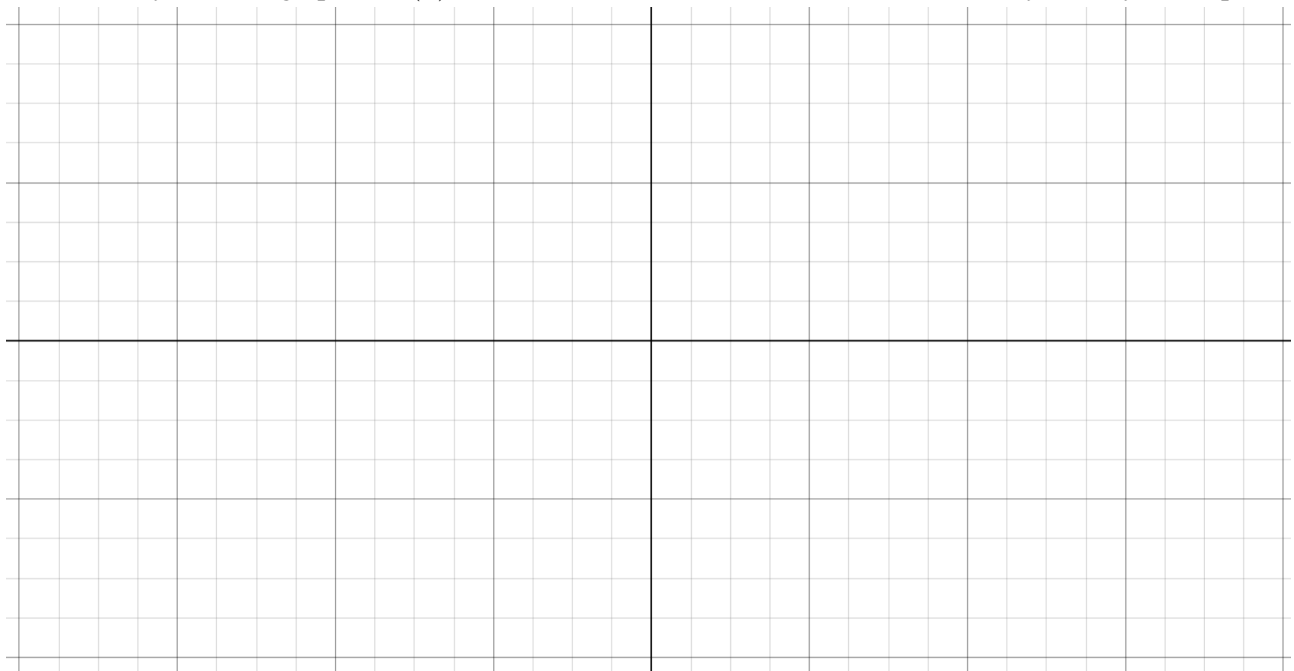
17. For $m(x) = 2 \cos \left(x - \frac{\pi}{4} \right)$ (10 pts)

(a) Identify the amplitude.

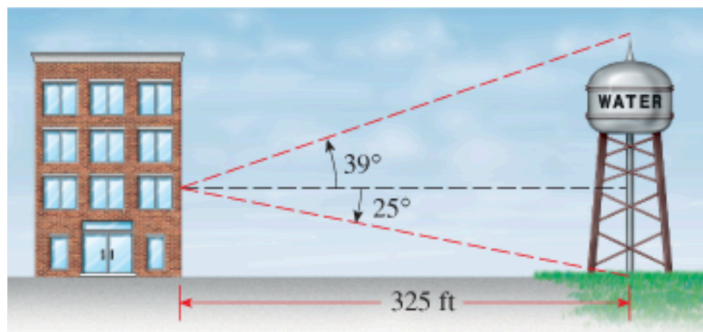
(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $m(x)$. Label at least two values on the x -axis and clearly identify the amplitude.



18. A water tower is located 325 ft from a building. From a window in the building, an observer notes that the angle of elevation to the top of the tower is 39° and that the angle of depression to the bottom of the tower is 25° . **Recall:** You don't have a calculator, so leave answers in exact form. (6 pts)



(a) How high is the **window**?

(b) How tall is the **tower**?