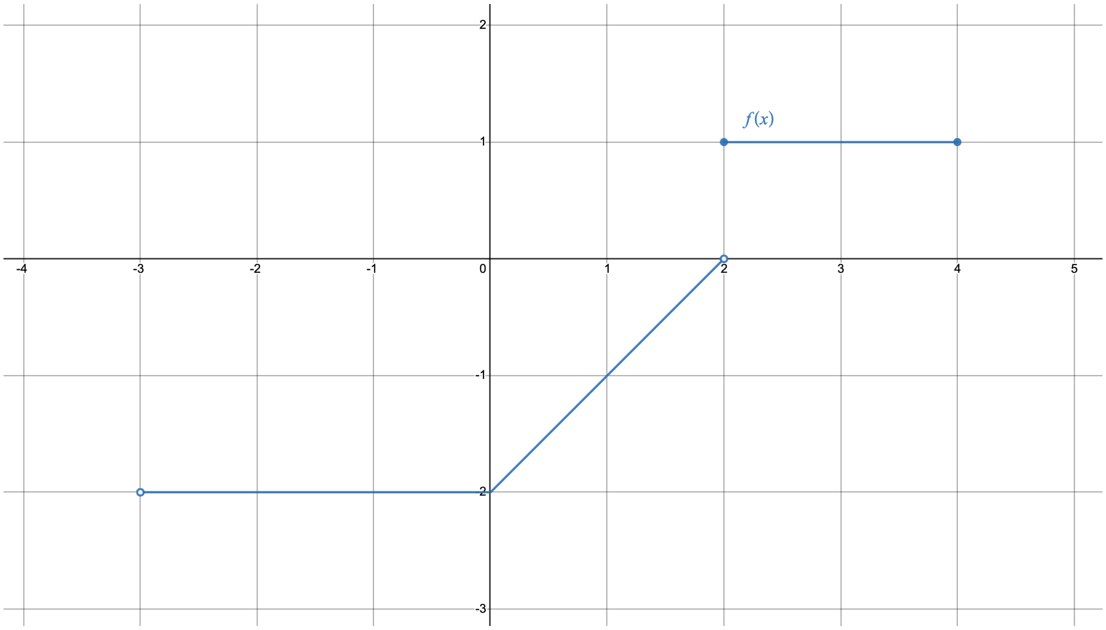
1. Answer the following for the given graph of a function $f(x)$. Give answers in interval notation where relevant (12 pts):



(a) Identify the domain of f.

Solution:

$\boxed{(-3, 4]}$

(b) Identify the the range of f.

Solution:

$\boxed{[-2, 0) \cup [1, 1]}$

(c) Find $(f + f)(-1)$.

Solution:

$(f + f)(-1) = f(-1) + f(-1) = -2 - 2 = \boxed{-4}$

(d) Find $f(-3)$ if it exists. If the value does not exist write "DNE."

Solution:

$f(-3):\ \boxed{DNE}$

(e) Solve $f(x) = -1$.

Solution:

$\boxed{x = 1}$

(f) Find $(f \circ f)(1)$.

Solution:

$(f \circ f)(1) = f(f(1)) = f(-1) = \boxed{-2}$.

(g) f is not one-to-one. Briefly explain why this function is not one-to-one.

Solution:

$\boxed{\text{The graph of } f \text{ does not pass the horizontal line test}}$. For example, $f(-2) = -2$ and $f(-1) = -2$ but $-2 \neq -1$.

(h) Find the x-values where $f(x) \geq 0$. Give your answer in interval notation.

Solution:

$\boxed{[2, 4]}$

(i) Find the net change of $f(x)$ from $x = 0$ to $x = 3$.

Solution:

$f(3) - f(0) = 1 - (-2) = \boxed{3}$

(j) Write down a piecewise-defined function that gives the same graph as $f(x)$.

Solution:

$$f(x) = \begin{cases} -2 & \text{if } \quad -3 < x \leq 0 \\ x - 2 & \text{if } \quad 0 < x < 2 \\ 1 & \text{if } \quad 2 \leq x \leq 4 \end{cases}$$

2. The following are unrelated. (7 pts)

(a) Find the center and radius of the circle that has equation: $x^2 + y^2 - 4y = 3$.

Solution:

$$\begin{aligned} x^2 + y^2 - 4y &= 3 \\ x^2 + y^2 - 4y + 4 &= 3 + 4 \\ x^2 + (y-2)^2 &= 7 \end{aligned}$$

$\boxed{\text{center: } (0, 2), \text{ radius: } \sqrt{7}}$

(b) Find the equation of the line that crosses through the points $(2, -3)$ and $(1, -1)$.

Solution:

The slope equation gives us $m = \dfrac{-1-(-3)}{1-2} = \dfrac{2}{-1} = -2$.

Using the slope-intercept equation, we get $y = -2x + b$. To solve for b we substitute in either point given. We will use the point $(2, -3)$.

$$\begin{aligned} y &= -2x + b && (1) \\ -3 &= -2(2) + b && (2) \\ -3 &= -4 + b && (3) \\ 1 &= b && (4) \end{aligned}$$

So we get the equation of the line $\boxed{y = -2x + 1}$.

3. Find the domain of the following functions. Express your answers in interval notation. (15 pts)

(a) $v(t) = 16t^2 + 64$

Solution:

There are no elements of the function that restrict the domain. So the domain is $\boxed{(-\infty, \infty)}$.

(b) $f(x) = \dfrac{\sqrt{x+2}}{x-4}$

Solution:

Since the square root of a negative number does not exist in the real numbers, then $x \geq -2$. However, $x - 4$ in the denominator cannot be zero, so $x \neq 4$. Thus the domain is $\boxed{[-2, 4) \cup (4, \infty)}$.

(c) $g(x) = \dfrac{x}{x^2 - 7x + 12}$

Solution:

Since the denominator cannot be zero, $x^2 - 7x + 12 \neq 0 \implies (x-3)(x-4) \neq 0$
$\implies x \neq 3, x \neq 4$. Thus the domain is $\boxed{(-\infty, 3) \cup (3, 4) \cup (4, \infty)}$.

4. For $f(x) = 2x^2 - 4$ compute the following for real number constant a and nonzero constant h: (6 pts)

(a) $f(a)$

Solution:

$\boxed{2a^2 - 4}$

(b) $f(a+h)$

Solution:

$$f(a+h) = 2(a+h)^2 - 4 \tag{5}$$
$$= \boxed{2a^2 + 4ah + 2h^2 - 4} \tag{6}$$

(c) $\dfrac{f(a+h) - f(a)}{h}$

Solution:

$$\frac{f(a+h) - f(a)}{h} = \frac{2(a+h)^2 - 4 - \left(2a^2 - 4\right)}{h} \tag{7}$$
$$= \frac{2a^2 + 4ah + 2h^2 - 4 - 2a^2 + 4}{h} \tag{8}$$
$$= \frac{4ah + 2h^2}{h} \tag{9}$$
$$= \frac{h\,(4a + 2h)}{h} \tag{10}$$
$$= \boxed{4a + 2h} \tag{11}$$

5. For $k(x) = \dfrac{1}{\sqrt{x}}$ and $j(x) = x^2 + 4$, find the following: (5 pts)

(a) Find $f(x) = (j \circ k)(x)$.

Solution:

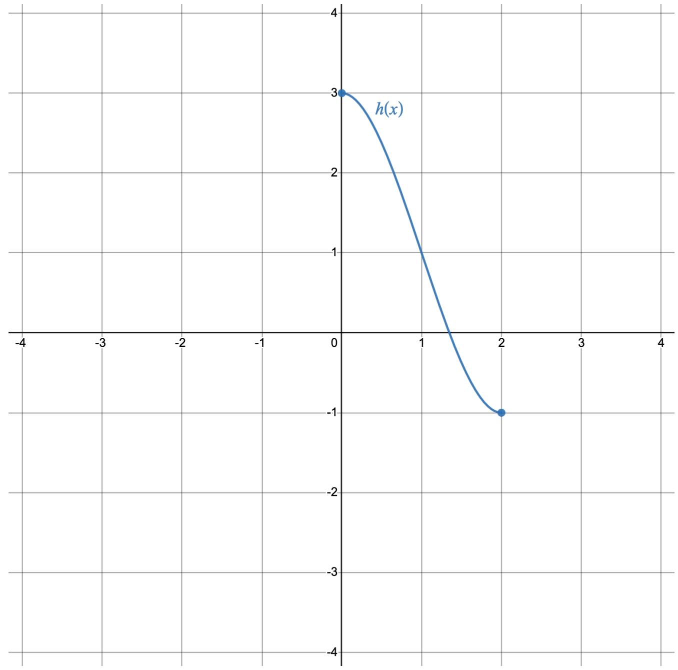
$$\begin{aligned}
(j \circ k)(x) &= j(k(x)) && (12)\\
&= j\left(\frac{1}{\sqrt{x}}\right) && (13)\\
&= \left(\frac{1}{\sqrt{x}}\right)^2 + 4 && (14)\\
&= \boxed{\frac{1}{x} + 4} && (15)
\end{aligned}$$

(b) Find the domain of $f(x)$.

Solution:

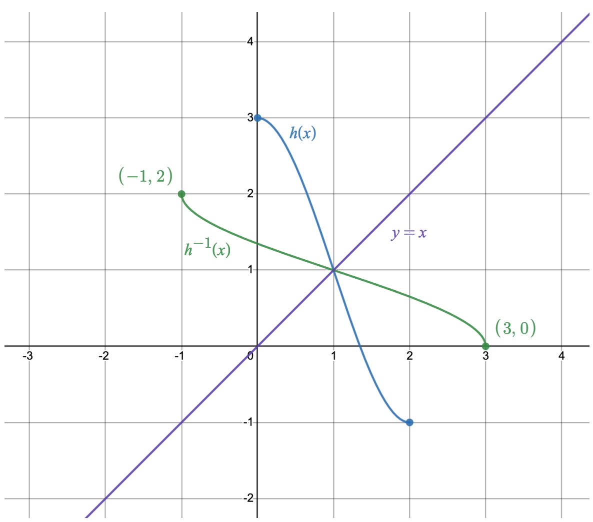
The domain of $f(x)$ must consider both $\dfrac{1}{\sqrt{x}}$ from line number 13 above and the final expression $\dfrac{1}{x} + 4$. Thus, the domain of $f(x)$ is $\boxed{(0, \infty)}$.

6. Answer the following for the one-to-one function $h(x)$ whose graph is given below with domain $[0, 2]$. (6 pts)



(a) On the graph to the right, graph the line $y = x$.

Solution:



(b) On the same graph sketch the graph of $h^{-1}(x)$ (label at least two points on the graph of $h^{-1}(x)$).

Solution:

See above graph.

(c) What is the range of $h^{-1}(x)$ in interval notation?

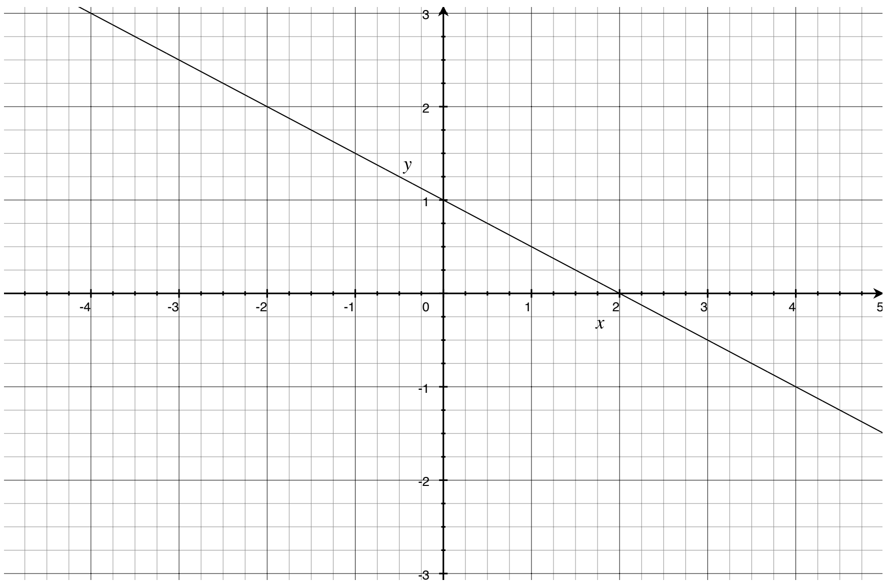
Solution:

The range of $h^{-1}(x)$ is the domain of $h(x)$ which is given in the statement of the problem. So the range of $h^{-1}(x)$ is $\boxed{[0, 2]}$.

7. Sketch the shape of the graph of each of the following on the provided axes. Make sure to label relevant value(s) on your axe(s) (19 pts)

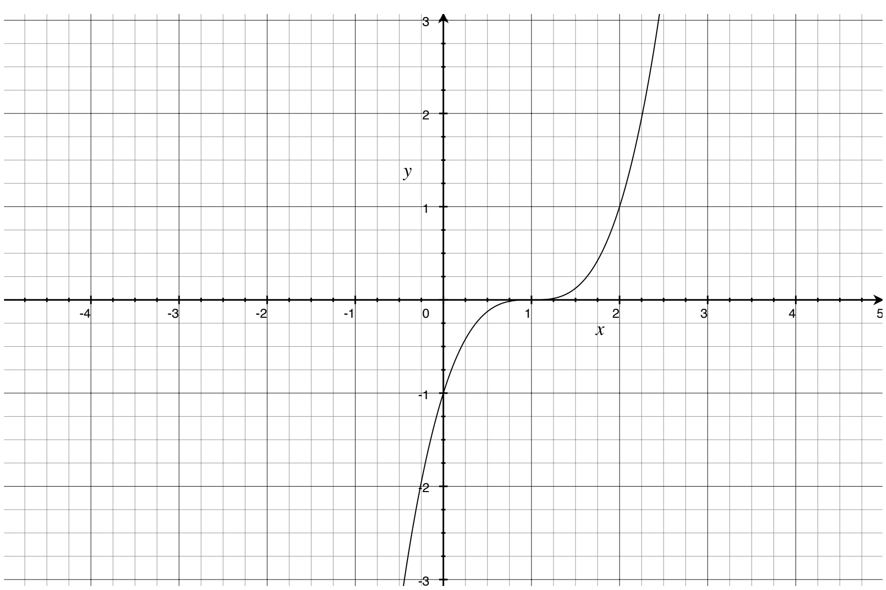
(a) $f(x) = -\frac{1}{2}x + 1$

Solution:



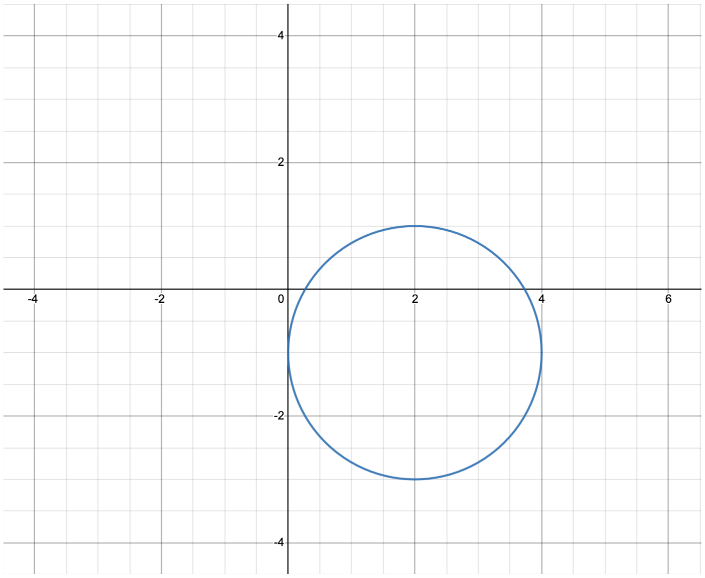
(b) $k(x) = (x - 1)^3$

Solution:



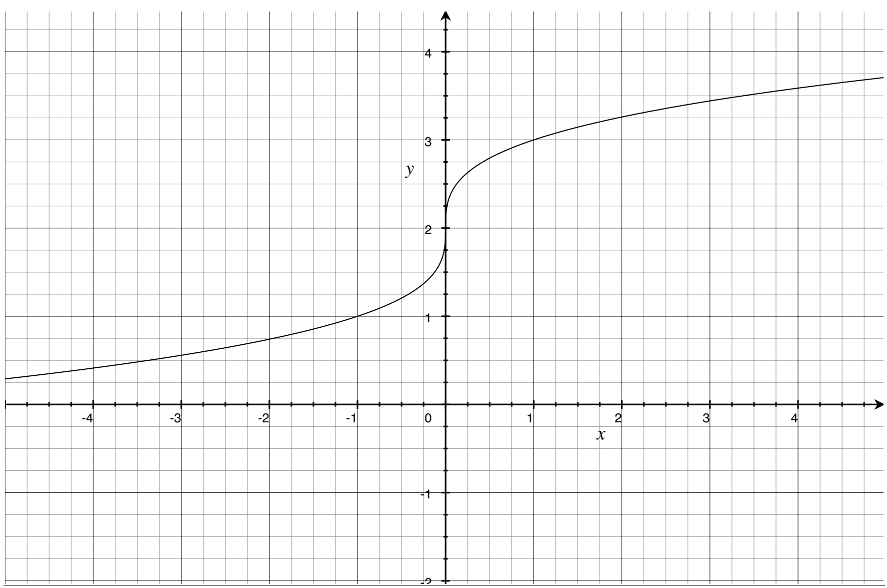
(c) $(x-2)^2 + (y+1)^2 = 4$

Solution:



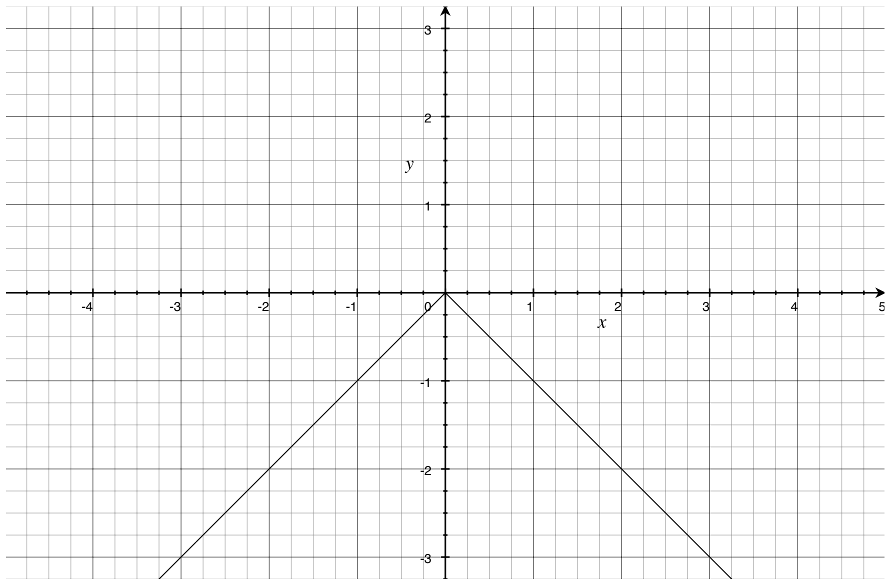
(d) $g(x) = \sqrt[3]{x} + 2$

Solution:



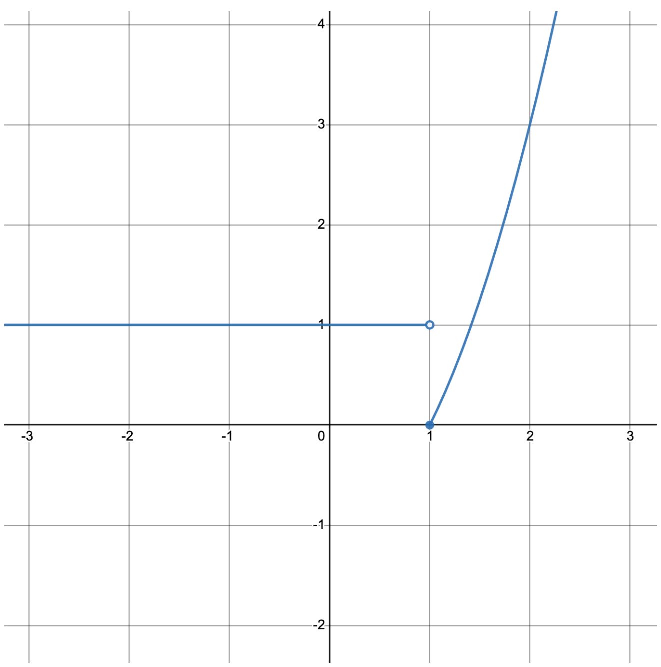
(e) $m(x) = -|x|$

Solution:



(f) $q(x) = \begin{cases} 1 & \text{if} \quad x < 1 \\ x^2 - 1 & \text{if} \quad x \geq 1 \end{cases}$

Solution:



8. For $P(x) = -x^4 - 5x^3 - 4x^2$ answer the following. (7 pts)

(a) Indicate on a graph or use arrow notation to indicate the end behavior of $P(x)$.

Solution:

For end behavior: $\boxed{P(x) \approx -x^4 \to -\infty \text{ as } x \to \infty \text{ and } P(x) \approx -x^4 \to -\infty \text{ as } x \to -\infty}$.

(b) Find the y-intercept of $P(x)$.

Solution:

The y-intercept is found by setting $x = 0$. So $P(0) = -4\left(0^4\right) - 5\left(0^3\right) - 4(0^2) = 0$. So the y-intercept is $\boxed{(0, 0)}$.

(c) Find all zeros and identify the multiplicity of each zero.

Solution:

The zeros of a polynomial are the x-values that result in $P(x) = 0$. So we set $-x^4 - 5x^3 - 4x^2 = 0$. By factoring:

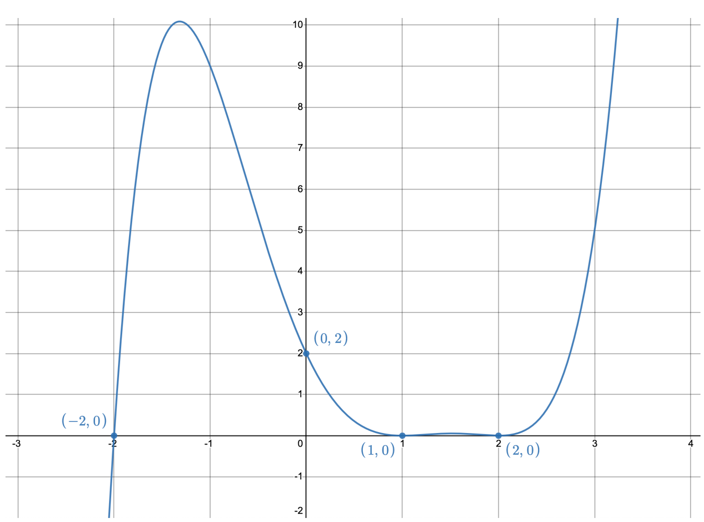
$$-x^4 - 5x^3 - 4x^2 = 0 \qquad (16)$$
$$-x^2\left(x^2 + 5x + 4\right) = 0 \qquad (17)$$
$$-x^2(x + 1)(x + 4) = 0 \qquad (18)$$

So we get $x = 0$ and $x = -1$ and $x = -4$ as the zeros. The multiplicity of $\boxed{x = 0}$ is $\boxed{2}$ and $\boxed{x = -1}$ is $\boxed{1}$ and $\boxed{x = -4}$ is $\boxed{1}$.

9. Sketch the shape of the graph of a polynomial function, $g(x)$, that satisfies **all** of the information. **Label** all intercepts on the graph. (5 pts)

i. The graph has y-intercept $(0, 2)$.
ii. The graph has end behavior consistent with $y = 2x^5$.
iii. The graph crosses at $(-2, 0)$ and bounces (touches but does not cross) at $(1, 0)$ and $(2, 0)$.
iv. The graph has no other x-intercepts.

Solution:



10. Use long division to find the quotient and remainder when $2x^3+3x^2-6x+2$ is divided by x^2-3. (5 pts)

Solution:

$$
\begin{array}{r l}
 & \;2x + \quad 3 \\
x^2-3 & \big)\; 2x^3 + 3x^2 - \quad 6x + \;2 \\
 & -(2x^3 \qquad\quad - \; 6x) \\
 & \qquad\qquad\qquad 3x^2 + \;2 \\
 & \qquad\qquad\; -\,(3x^2 - \;\;9\,) \\
 & \qquad\qquad\qquad\qquad\; 11
\end{array}
$$

So the quotient is $\boxed{2x+3}$ and the remainder is $\boxed{11}$.

11. The following are unrelated. (6 pts)

(a) Is $f(x) = x^6 - |x| + 1$ odd, even, or neither? Justify your answer to earn credit.

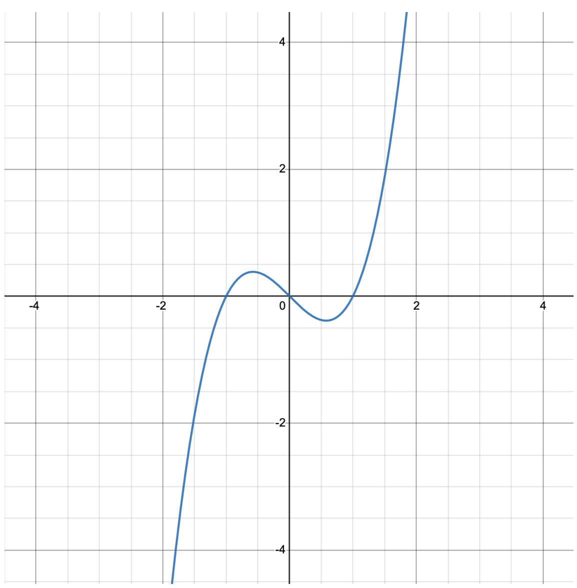
Solution:

Replacing x by $-x$ and using the fact that $(-x)^6 = x^6$ and $|-x| = |x|$ we get:

$$
\begin{aligned}
f(-x) = &= (-x)^6 - |-x| + 1 && (19) \\
&= x^6 - |x| + 1 && (20) \\
&= f(x) && (21)
\end{aligned}
$$

Since $f(-x) = f(x)$ then $f(x)$ is $\boxed{\text{even}}$.

(b) Is the graph below that of an odd function, even function, or neither?

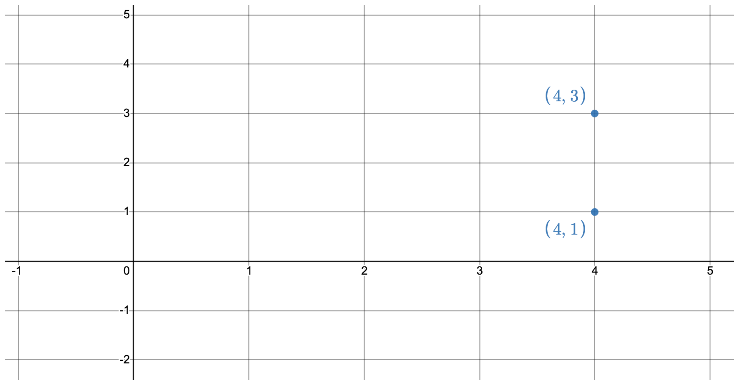


Solution:

The graph is symmetric about the origin and is thus $\boxed{\text{odd}}$.

12. (a) Plot the points $C(4, 1)$ and $D\,(4, 3)$ on the graph below. (7 pts)

Solution:



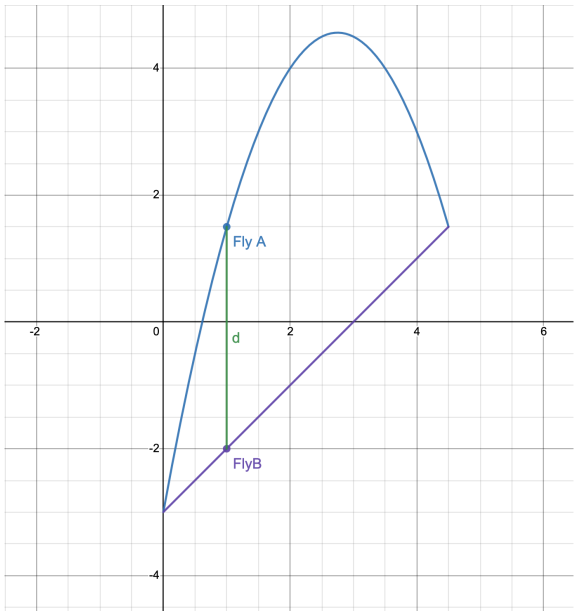
(b) Find the distance between points C and D.

Solution:

The distance between the two points can be found using the distance formula: $d = \sqrt{(4-4)^2 + (3-1)^2} = \boxed{2}$.

Note that this is the same as simply subtracting the smaller y-coordinate from the larger y-coordinate: $3 - 1 = 2$.

(c) Two flies, Fly A and Fly B, are crawling along a wall in such a way that Fly A is always directly above Fly B (See picture). Fly A is crawling along the path $f(x) = -x^2 + \frac{11}{2}x - 3$ and fly B is crawling along path $g(x) = x - 3$. Find the maximal distance, d, between the two flies on the interval of x-values: $[0, 4.5]$. As always, show all work in justifying your answer.



Solution:

The vertical distance between the two flies can be found by subtracting the smaller y-coordinate from the larger y-coordinate. Letting d represent the distance between the two flies we get:

$$d = -x^2 + \frac{11}{2}x - 3 - (x - 3) \tag{22}$$

$$= -x^2 + \frac{9}{2}x \tag{23}$$

The x-coordinate, where the maximum distance is located, can be found by using the vertex formula $h = \dfrac{-b}{2a}$ where $a = -1$ and $b = \dfrac{9}{2}$. Thus $h = \dfrac{-\left(\frac{9}{2}\right)}{2(-1)} = \dfrac{9}{4}$. The maximum distance is found by plugging in h: $d = -\left(\dfrac{9}{4}\right)^2 + \dfrac{9}{2}\left(\dfrac{9}{4}\right) = -\dfrac{81}{16} + \dfrac{81}{8} = \boxed{\dfrac{81}{16}}$.