1. The following are unrelated: (18 pts)

(a) Rewrite each of the following without the absolute value symbol:

i. $|\pi - 3|$

Solution:

Since $\pi > 3$, $\pi - 3 > 0$ so $|\pi - 3| = \boxed{\pi - 3}$

ii. $|x - 2|$ where $x < 2$

Solution:

Since $x < 2$, $x - 2 < 0$ and therefor $|x - 2| = \boxed{-(x-2)}$ or $\boxed{2 - x}$

(b) Divide/Add as indicated: $-\frac{\frac{6}{5}}{\frac{8}{3}} + 10^{-1}$

Solution:

$$-\frac{\frac{6}{5}}{\frac{8}{3}} + 10^{-1} = -\left(\frac{3}{8}\right)\left(\frac{6}{5}\right) + \frac{1}{10} \tag{1}$$
$$= -\frac{9}{20} + \frac{2}{20} \tag{2}$$
$$= \boxed{-\frac{7}{20}} \tag{3}$$

(c) Let x, y, and z be real numbers such that $x > 3$, $y < 0$, and $1 \leq z \leq 3$. Answer the following:

i. Is $x^7 y^{24}$ positive, negative, or cannot be determined? No work is needed to justify your answer.

Solution:

$\boxed{\text{Positive}}$

ii. Is $z - 2 + y$ positive, negative, or cannot be determined? No work is needed to justify your answer.

Solution:

$\boxed{\text{Cannot be determined}}$

iii. Is $-z^4 y^3$ positive, negative, or cannot be determined? No work is needed to justify your answer.

Solution:

$\boxed{\text{Positive}}$

(d) Perform the multiplication and subtraction and simplify: $(x-2)^2 - 3\left(x^2+4\right)$.

Solution: $= -2x^2 - 4x - 8 = (-2)\left(x^2+2x+4\right)$

$$\begin{aligned}(x-2)^2 - 3\left(x^2+4\right) &= x^2 - 4x + 4 - \left(3x^2+12\right) &(4)\\ &= \boxed{-2x^2-4x-8} &(5)\end{aligned}$$

2. The following are unrelated: (24 pts)

(a) Simplify the expression: $\sqrt{45}$

Solution:

$$\begin{aligned}\sqrt{45} &= \sqrt{3^2\cdot 5} &(6)\\ &= \boxed{3\sqrt{5}} &(7)\end{aligned}$$

(b) Simplify the expression: $\sqrt{2\sqrt{16}}$

Solution:

$$\begin{aligned}\sqrt{2\sqrt{16}} &= \sqrt{2\sqrt{2^4}} &(8)\\ &= \sqrt{2\cdot 2^2} &(9)\\ &= \boxed{2\sqrt{2}} &(10)\end{aligned}$$

(c) Rewrite with positive exponents and simplify: $\left(x^2+1\right)^{-1}\left(x^2+1\right)^{-4}$

Solution:

$$\begin{aligned}\left(x^2+1\right)^{-1}\left(x^2+1\right)^{-4} &= \frac{1}{\left(x^2+1\right)}\cdot\frac{1}{\left(x^2+1\right)^4} &(11)\\ &= \boxed{\frac{1}{\left(x^2+1\right)^5}} &(12)\end{aligned}$$

(d) Simplify (Give your answer without negative exponents): $\left(-3b^5\right)\left(2c^5b^2a^{-3}\right)^2$

Solution:

$$\begin{aligned}\left(-3b^5\right)\left(2c^5b^2a^{-3}\right)^2 &= -3b^5\left(2^2c^{10}b^4a^{-6}\right) &(13)\\ &= \boxed{-\frac{12b^9c^{10}}{a^6}} &(14)\end{aligned}$$

(e) Multiply: $\left(\sqrt{x-4}-2\right)^2$

Solution:

$$\begin{aligned}\left(\sqrt{x-4}-2\right)^2 &= \left(\sqrt{x-4}-2\right)\left(\sqrt{x-4}-2\right) &(15)\\ &= (x-4) + \sqrt{x-4}\,(-2) + (-2)\sqrt{x-4} + (-2)^2 &(16)\\ &= (x-4) - 4\sqrt{x-4} + 4 &(17)\\ &= \boxed{x - 4\sqrt{x-4}} &(18)\end{aligned}$$

(f) Multiply: $x^8\left(x^{1/2}-\frac{3}{x^4}\right)$

Solution:

$$x^8\left(x^{1/2}-\frac{3}{x^4}\right) = x^8x^{1/2}-\frac{3x^8}{x^4} \tag{19}$$

$$= \boxed{x^{17/2}-3x^4} \tag{20}$$

3. The following are unrelated: (18 pts)

(a) Evaluate $18(-x)^3$ for $x=-\frac{1}{3}$

Solution:

$$18\left(-\left(-\frac{1}{3}\right)\right)^3 = 18\left(\frac{1}{3}\right)^3 \tag{21}$$

$$= 18\cdot\frac{1}{27} \tag{22}$$

$$= \boxed{\frac{2}{3}} \tag{23}$$

(b) Factor completely (If not factorable write NF): $8x^3+1$

Solution:

$8x^3+1$ can be factored as a sum of two cubes, using the formula provided on exam 1: $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$. Letting $a=2x$ and $b=1$:

$$8x^3+1=(x+1)\left((2x)^2-2x+1\right) \tag{24}$$

$$= \boxed{(x+1)\left(4x^2-2x+1\right)} \tag{25}$$

Note that $4x^2-2x+1$ does not factor any further.

(c) Multiply: $\frac{x^2-16}{x^2-5x+6}\cdot\frac{x^3-2x^2}{3x-12}$

Solution:

We start by writing under one fraction bar and factoring to see if anything cancels:

$$\frac{x^2-16}{x^2-5x+6}\cdot\frac{x^3-2x^2}{3x-12} = \frac{\left(x^2-16\right)\left(x^3-2x^2\right)}{\left(x^2-5x+6\right)(3x-12)} \tag{26}$$

$$= \frac{(x-4)(x+4)\,x^2\,(x-2)}{(x-3)(x-2)\,3\,(x-4)} \tag{27}$$

$$= \boxed{\frac{(x+4)\,x^2}{(x-3)\,3}} \tag{28}$$

(d) Simplify the compound fraction: $\dfrac{\frac{5}{x^2} - \frac{1}{x}}{1 - \frac{25}{x^2}}$

Solution:

We start by getting a single fraction in the numerator and denominator we we carry out the division:

$$
\begin{aligned}
\frac{\frac{5}{x^2} - \frac{1}{x}}{1 - \frac{25}{x^2}} &= \frac{\frac{5}{x^2} - \frac{x}{x^2}}{\frac{x^2}{x^2} - \frac{25}{x^2}} && (29)\\
&= \frac{\frac{5-x}{x^2}}{\frac{x^2-25}{x^2}} && (30)\\
&= \frac{5-x}{x^2} \cdot \frac{x^2}{x^2-25} && (31)\\
&= -\frac{x-5}{x^2-25} && (32)\\
&= -\frac{x-5}{(x-5)(x+5)} && (33)\\
&= \boxed{-\frac{1}{x+5}} && (34)
\end{aligned}
$$

4. Solve the following equation over the complex numbers: $z^2 - 4z + 5 = 0$. (4 pts)

Solution:

We start by trying to factor the left hand side. Unfortunately, the left hand side does not factor easily, so we utilize the quadratic formula.

$$
\begin{aligned}
z &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)} && (35)\\
&= \frac{4 \pm \sqrt{-4}}{2} && (36)\\
&= \frac{4 \pm 2i}{2} && (37)\\
&= \frac{2(2 \pm i)}{2} && (38)\\
&= \boxed{2 \pm i} && (39)
\end{aligned}
$$

5. Solve each of the following equations: (12 pts)

(a) $\frac{1}{3}x - \frac{5}{6} = \frac{1}{2}x - 1$

Solution:

We start by clearing the fractions by multiplying both sides of the equation by 6.

$$\frac{1}{3}x - \frac{5}{6} = \frac{1}{2}x - 1 \tag{40}$$
$$6\left(\frac{1}{3}x - \frac{5}{6}\right) = 6\left(\frac{1}{2}x - 1\right) \tag{41}$$
$$6\left(\frac{1}{3}x\right) - 6\left(\frac{5}{6}\right) = 6\left(\frac{1}{2}x\right) - 6(1) \tag{42}$$
$$2x - 5 = 3x - 6 \tag{43}$$
$$1 = x \tag{44}$$
$$x = \boxed{1} \tag{45}$$

(b) $x^3 - 18x = 17x^2$

Solution:

We start by moving all terms to one side and then factoring.

$$x^3 - 18x = 17x^2 \tag{46}$$
$$x^3 - 17x^2 - 18x = 0 \tag{47}$$
$$x(x-18)(x+1) = 0 \tag{48}$$

By the multiplicative property of zero, we set $x = 0$, $x - 18 = 0$ and $x + 1 = 0$ resulting in solutions $\boxed{x = 0, x = 18, \text{ and } x = -1}$.

(c) $\sqrt{3x+3} - 2 = x - 1$

We start by isolating the square root so we can square both sides. We then can solve the equation by factoring.

$$\sqrt{3x+3} - 2 = x - 1 \tag{49}$$
$$\sqrt{3x+3} = x + 1 \tag{50}$$
$$3x + 3 = (x+1)^2 \tag{51}$$
$$3x + 3 = x^2 + 2x + 1 \tag{52}$$
$$0 = x^2 - x - 2 \tag{53}$$
$$0 = (x-2)(x+1) \tag{54}$$

This results in two potential solutions: $x = 2$ and $x = -1$. We must check both values:

Checking $x = 2$ we get $\sqrt{3(2)+3} - 2 = (2) - 1 \implies \sqrt{9} - 2 = 1 \implies 3 - 2 = 1$ which is a solution.

Checking $x = -1$ we get $\sqrt{3(-1)+3} - 2 = (-1) - 1 \implies \sqrt{0} - 2 = -2 \implies -2 = -2$ which is also a solution.

So the solution is: $\boxed{x = 2 \text{ and } x = -1}$.

6. Solve each of the following equations: (12 pts)

(a) Solve for m: $T = mg + ma$

Solution:

The goal is to get m by itself on one side of the equation.

$$T = mg + ma \tag{55}$$
$$T = m(g + a) \tag{56}$$
$$\frac{T}{g+a} = m \tag{57}$$

So the answer is $\boxed{m = \dfrac{T}{g+a}}$

(b) $-\frac{1}{x^2-x} - \frac{2}{x^2-1} = \frac{2}{x^2+x}$

Solution:

We start by factoring each denominator to find the lowest common denominator and then we multiply each side of the equation by the lowest common denominator found.

$$-\frac{1}{x^2-x} - \frac{2}{x^2-1} = \frac{2}{x^2+x} \tag{58}$$

$$-\frac{1}{x(x-1)} - \frac{2}{(x-1)(x+1)} = \frac{2}{x(x+1)} \tag{59}$$

$$x(x-1)(x+1) \cdot \left(-\frac{1}{x(x-1)} - \frac{2}{(x-1)(x+1)}\right) = x(x-1)(x+1) \cdot \left(\frac{2}{x(x+1)}\right) \tag{60}$$

$$-(x+1) - 2x = 2(x-1) \tag{61}$$

$$-x - 1 - 2x = 2x - 2 \tag{62}$$

$$1 = 5x \tag{63}$$

$$\frac{1}{5} = x \tag{64}$$

Resulting in one potential solution: $x = \frac{1}{5}$. Plugging in $\boxed{x = \frac{1}{5}}$, we see that it is in fact a solution.

(c) $|x+2| = 5$

Solution:

The x-values that solve $|x+2| = 5$ are found when we set $x+2=5$ and $x+2=-5$. This results in two solutions: $\boxed{x=3}$ and $\boxed{x=-7}$.

7. Solve the following inequalities. Justify your answers by using a number line or sign chart if needed. Answers without full justification will not receive full credit. Express all answers in interval notation. (8 pts)

(a) $2x^2 - 4x < 6$

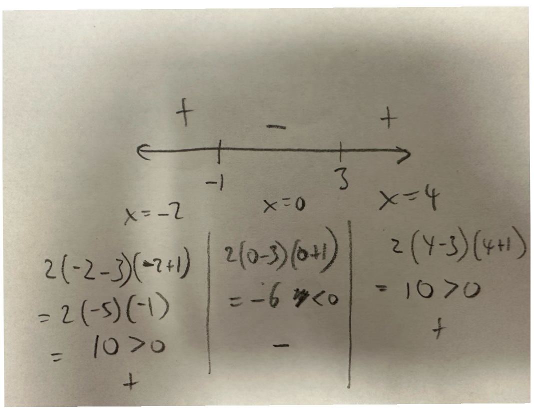
Solution:

We start by adding 6 to both sides to get a zero on the right side and then we factor.

$$2x^2 - 4x < 6 \tag{65}$$
$$2x^2 - 4x - 6 < 0 \tag{66}$$
$$2(x^2 - 2x - 3) < 0 \tag{67}$$
$$2(x-3)(x+1) < 0 \tag{68}$$

Setting the left side equal to zero we get two values that make the left side zero: $x = -1$ and $x = 3$.

Placing these on a number line and picking test values we get



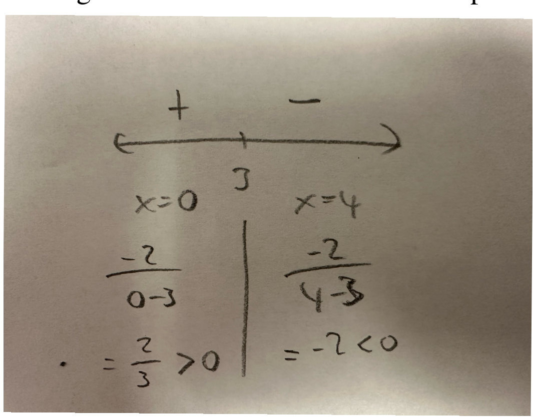
and the solution $\boxed{(-1, 3)}$.

(b) $\dfrac{-2}{x-3} \geq 0$

Solution:

We start by noting that the sign of the left side can only change when $x - 3 = 0$ or when $x = 3$.

Placing this value on a number line and picking test values we get



and the solution $\boxed{(-\infty, 3)}$. NOTE: We do not include 3 in the answer because this value does not solve the inequality.

8. Suppose you know that a is a real number and that $\left(a - \frac{1}{2}i\right)\left(a + \frac{1}{2}i\right) = 3$. Find all possible value(s) of a that make this equality true. (4 pts)

Solution:

We start by multiplying out the left side and then we can solve for a:

$$\left(a - \frac{1}{2}i\right)\left(a + \frac{1}{2}i\right) = 3 \tag{69}$$

$$a^2 + \frac{1}{2}ai - \frac{1}{2}ai - \frac{1}{4}i^2 = 3 \tag{70}$$

$$a^2 - \frac{1}{4}(-1) = 3 \tag{71}$$

$$a^2 + \frac{1}{4} = 3 \tag{72}$$

$$a^2 = 3 - \frac{1}{4} \tag{73}$$

$$a^2 = \frac{12}{4} - \frac{1}{4} \tag{74}$$

$$a^2 = \frac{11}{4} \tag{75}$$

$$a = \pm\sqrt{\frac{11}{4}} \tag{76}$$

$$a = \boxed{\pm\frac{\sqrt{11}}{2}} \tag{77}$$