INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources), help from another person, are not permitted during the exam. **Give all answers in exact form.**

Potentially useful formulas:

1. $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

NOTE: YOU MAY TEAR OFF THIS PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.
ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE
iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE.
iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND EXIT TO THE EXAM SUBMISSION AREA.

Scratch Paper

Name: ____

1. The following are unrelated. Simplify answers and leave without negative exponents. (16 pts)

   (a) Evaluate and simplify: $\sqrt{24}+\frac{1}{2}-\frac{5}{6}-\frac{\frac{8}{3}}{2}$

   (b) Simplify: $-\left(x^2-1\right)^2-\left(-x^3\right)^2\left(\frac{x^{-3}}{x}\right)$

   (c) Multiply and simplify: $\frac{x^2+x+1}{x^3-16x}\cdot\frac{x-4}{x^3-1}$

   (d) Simplify: $\sqrt{x}\left(\sqrt{x}-x^{2/3}\right)+\frac{1}{\left(x^{-1/9}\right)^3}$

2. The following are unrelated. Simplify answers and leave without negative exponents. (8 pts)

   (a) Simplify: $\dfrac{\frac{3}{x}+\frac{3}{x^2}}{\frac{1}{x}+1}$

   (b) Simplify: $\log_4(64)-\log_5\left(5^3\right)+\log_4(32)-\log_4(2)+\log(1)$ (Your answer should have no logarithms)

3. Find all real and complex solutions: $x^3-x^2+x=0$. Give complex solutions in $a+bi$ form. (4 pts)

4. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (16 pts)

(a) Solve for n: $\dfrac{3}{n+1} = \dfrac{1}{2n}$

(b) Solve for r: $U - U_\circ = \dfrac{kq_1q_2}{r^2}$

(c) Solve for x: $3 + 2\log_4(x) = 1$

(d) Solve for t: $7e^{-0.2t} = 12$

5. For the two points $P(-3, 1)$ and $Q(-1, 4)$: (11 pts)

   (a) Find the slope of the line through the two points.

   (b) Find the equation of the line that passes through the points P and Q.

   (c) Find the equation of the circle centered at Q that passes through P.

6. Consider the functions: $m(x) = \sqrt{x-3}$, $r(x) = x^2 - 2$, and $q(x) = e^{2x}$. (12 pts)

   (a) Find the domain of $m(x)$. Give your answer in interval notation.

   (b) Find $(r \circ m)(x)$.

   (c) Find the domain of $(r \circ m)(x)$. Give your answer in interval notation.

   (d) Find $q(\ln(2))$.

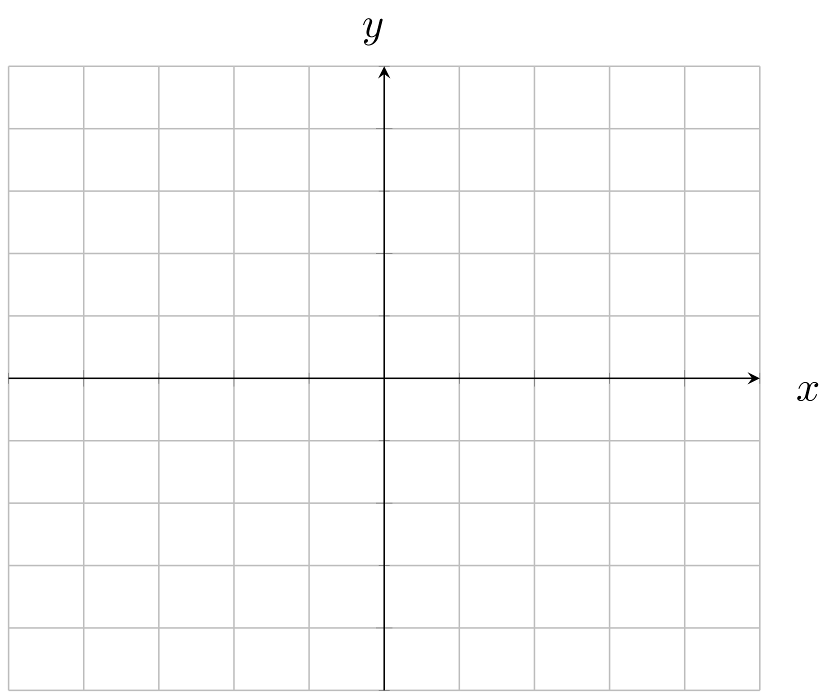
7. For $P(x) = x^4 - x^3 - 6x^2$ answer the following (13 pts):

   (a) Find the x-intercept(s).

   (b) Find the y-intercept.

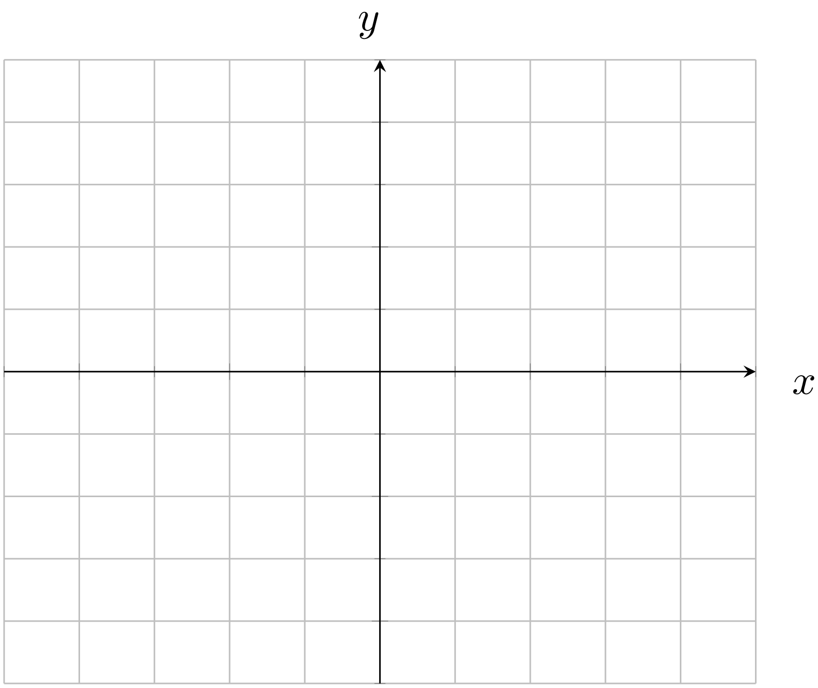
   (c) Indicate on a graph or use arrow notation to indicate the end behavior of $P(x)$.

   (d) Graph $P(x)$.



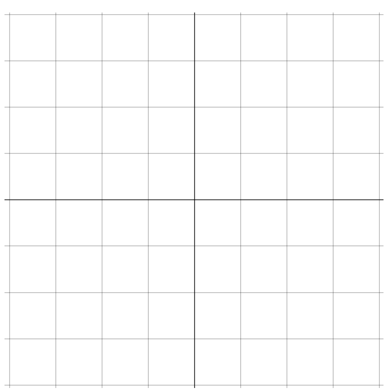
8. Sketch the graph of the rational function, $y = R(x)$, with the following properties: (5 pts)

   i. The graph of $y = R(x)$ has a slant asymptote of $y = x - 1$.
   ii. The graph of $y = R(x)$ crosses the x-intercept $(2, 0)$.
   iii. The graph of $y = R(x)$ has vertical asymptote $x = -2$.
   iv. The graph has no other x-intercepts.

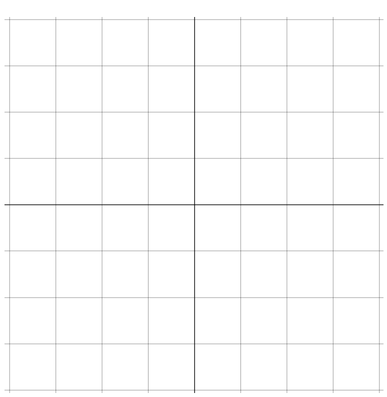


9. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (14 pts)

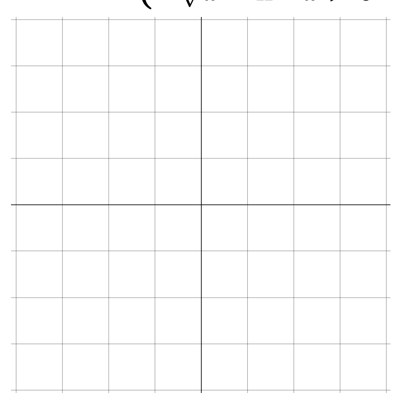
(a) $f(x) = |x + 1|$.



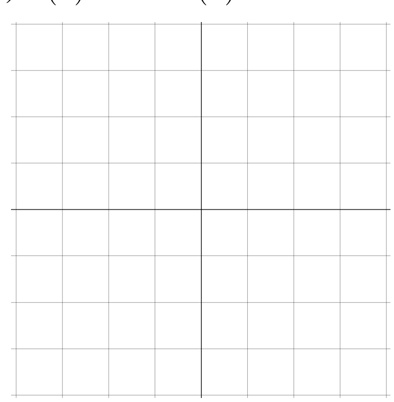
(b) $h(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



(c) $q(x) = \begin{cases} e^x & \text{if} \quad x < 0 \\ 2 & \text{if} \quad x = 0 \\ -\sqrt{x} & \text{if} \quad x > 0 \end{cases}$



(d) $k(x) = \tan^{-1}(x)$



10. Find the exact value: (18 pts)

(a) $\sin\left(\frac{2\pi}{3}\right)$

(d) $\cos\left(\sin^{-1}(1)\right)$

(b) $\cos\left(-\frac{\pi}{4}\right)$

(e) $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

(c) $\arctan\left(\frac{1}{\sqrt{3}}\right)$

(f) $\cos\left(\frac{3\pi}{8}\right)$

11. Verify the identity: $\dfrac{\sec(\theta)\cos^2(\theta)+\sin(\theta)\tan(\theta)}{\sec(\theta)}=1$. (5 pts)

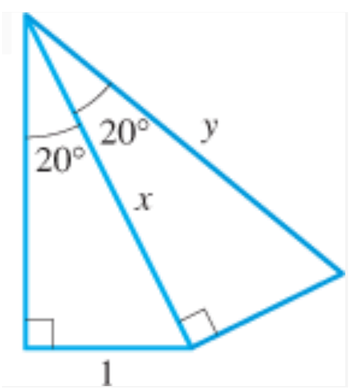
12. Is $f(x)=\sin^2(x)\cos(x)$ odd, even, or neither? Justify your answer for credit. (4 pts)

13. Find all solutions to the following equations: (8 pts)

(a) $\sin(\theta) - \tan(\theta)\sin(\theta) = 0$

(b) $\cos\left(\frac{\theta}{3}\right) = \frac{1}{2}$

14. Solve for both x and y. (6 pts)



15. For $f(x) = 2\sin\left(x - \frac{\pi}{3}\right)$ (10 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the horizontal shift.

(d) Sketch one cycle of the graph of $f(x)$. Be sure to label at least two values on the x axis and clearly identify the amplitude.