INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources) are not permitted. **Give all answers in exact form.**

Potentially useful formulas:

$$\log_b(B) = \frac{\log_a(B)}{\log_a(b)} \text{ for } a > 0, a \neq 1.$$

$$A = \frac{1}{2}r^2\theta$$

$$S = r\theta$$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

    i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

    ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

    iii. WRITE YOUR NAME ON THE NEXT PAGE.

    iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND LINE UP AT THE BACK OF THE ROOM. A PROCTOR WILL INDICATE WHEN IT'S YOUR TURN TO EXIT THE ROOM AND UPLOAD TO GRADESCOPE.

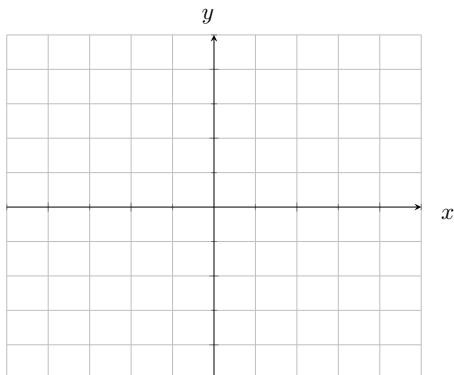1. For $R(x) = \dfrac{x^2 - 16}{2x^2 - 2x - 24}$ (9 pts)

    (a) Find the location $(x, y$ -coordinates) of any hole(s). If there are none state NONE.

    (b) Find any horizontal or slant asymptote(s). If there are none state NONE.
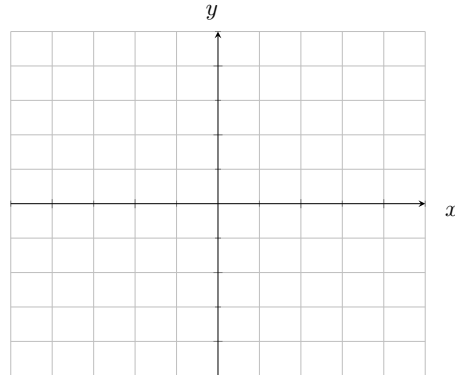
    (c) Find any vertical asymptote(s). If there are none state NONE.

2. Sketch the following graphs: Be sure to label any asymptotes and intercepts for each graph.

    (a) $q(x) = -e^x$ (3 pts)   (b) $f(x) = \log_3(x + 1)$ (3 pts)

3. Simplify: $2^x \left(2^x - 2^{-x}\right) - \left(2^x\right)^2$. (3 pts)

4. (a) Simplify (rewrite without logs): $-\log_3(27) + \log_9(9^{\sqrt{2}}) + \log_5\left(\dfrac{1}{\sqrt{5}}\right) - e^{\ln(3)}$ (4 pts)

(b) Rewrite as a single logarithm without negative exponents: $3\ln\left(z^2\right) - 4\ln(z) - \dfrac{2}{3}\ln(z)$ (4 pts)

5. Solve the following equations for x. If there are no solutions write "no solutions" (be sure to justify answer for full credit).

(a) $10 = 4e^{-2x}$ (4 pts)

(b) $7 = 2^{3+2x}$ (4 pts)

6. Solve the following equations for x. If there are no solutions write "no solutions" (be sure to justify answer for full credit).

(a) $3^{5x+1} = 9^x$ (4 pts)

(b) $\log(2x) = \log\left(3x^2 - 9\right) - \log(3)$ (4 pts)

(c) $5 = 2\log_2\left(x - \dfrac{1}{2}\right)$ (4 pts)

7. The raccoon population in a certain Colorado town was 1000 in 2022, and the population is expected to double every 5 years.
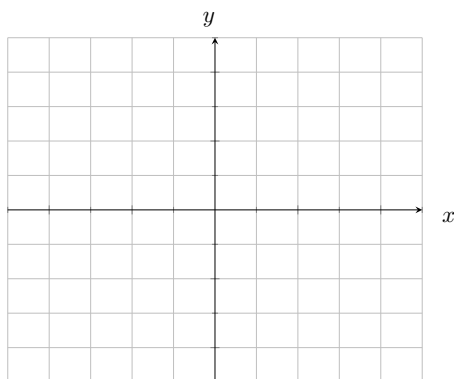
   (a) Find an exponential model $n(t) = n_\circ 2^{t/a}$ for the number of raccoons after t years. (3 pts)

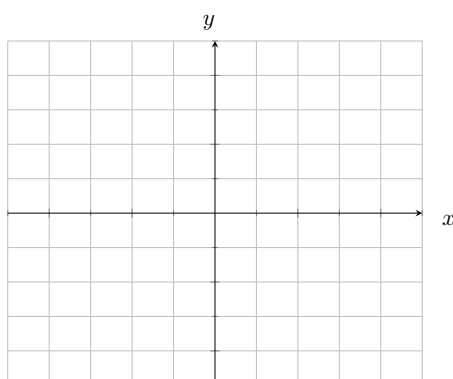   (b) What is the expected population size in 2032? (3 pts)

   (c) How many years until the population triples? (3 pts)

8. Sketch each angle in standard position on the unit circle.

   (a) $\theta = \dfrac{4\pi}{3}$ (2 pts)

   (b) $\theta = -\dfrac{5\pi}{4}$ (2 pts)

9. The following are unrelated.

(a) For an angle θ in standard position, suppose we know $\cos(\theta) > 0$ and $\sin(\theta) < 0$. What quadrant does the terminal side of θ lie? (3 pts)

(b) Find an angle between 0 and 2π that is co-terminal with $\theta = \dfrac{7\pi}{2}$ (3 pts)

(c) Find $\cos t$ in terms of $\sin t$ if the terminal point determined by t is in quadrant II. (3 pts)

10. Answer the following for $\tan(\theta) = \dfrac{3}{5}$.

(a) Sketch a triangle that has acute angle θ (3 pts).

(b) Find $\cos \theta$ (3 pts)

(c) Find $\sec \theta$ (3 pts)

11. Find the exact value of each of the following. If a value does not exist write DNE.

(a) $\sin(180°)$ (3 pts)
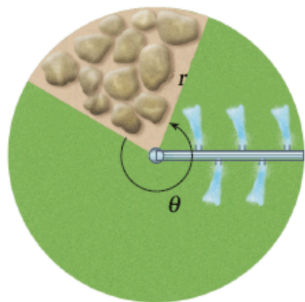
(b) $\cos\left(\dfrac{2\pi}{3}\right)$ (3 pts)

(c) $\tan\left(-\dfrac{5\pi}{6}\right)$ (3 pts)

(d) $\csc(240°)$ (3 pts)

(e) $\cos\left(\dfrac{7\pi}{4}\right)$ (3 pts)

12. An irrigation system uses a straight sprinkler pipe $r = 6$ ft long that pivots around a central point as shown. Because of an obstacle the pipe is allowed to pivot through $\theta = 260°$ only. Find the area irrigated by this system. (4 pts)

13. A 14-ft ladder leans against a building so that the angle between the ground and the ladder is $60°$. How high does the ladder reach on the building? (4 pts)