INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from another person are not permitted while taking the exam. The exam is worth 100 points.

Potentially useful formulas:

(i) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

(ii) $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

1. The following are unrelated: (18 pts)

(a) Add/Subtract as indicated: $2^0 + \frac{3}{8} - \frac{1}{12} + 6^{-1}$

Solution:

$$\begin{aligned} 2^0 + \frac{3}{8} - \frac{1}{12} + 6^{-1} &= 1 + \frac{3}{8} - \frac{1}{12} + \frac{1}{6} \\ &= \frac{24}{24} + \frac{9}{24} - \frac{2}{24} + \frac{4}{24} \\ &= \boxed{\frac{35}{24}} \end{aligned}$$

(b) Evaluate the expression: $\frac{|7-9|}{2-|-3|}$

Solution:

$$\begin{aligned} \frac{|7-9|}{2-|-3|} &= \frac{|-2|}{2-3} \\ &= \frac{2}{-1} = \boxed{-2} \end{aligned}$$

(c) Evaluate the expression: $\sqrt{27}$

Solution:

$$\begin{aligned} \sqrt{27} &= \sqrt{9 \cdot 3} \\ &= \boxed{3\sqrt{3}} \end{aligned}$$

(d) Evaluate the expression: $\sqrt{6}\sqrt{15}$

Solution:

$$\begin{aligned} \sqrt{6}\sqrt{15} &= \sqrt{2 \cdot 3}\sqrt{3 \cdot 5} \\ &= \sqrt{2}\sqrt{3}\sqrt{3}\sqrt{5} \\ &= 3\sqrt{2}\sqrt{5} \\ &= \boxed{3\sqrt{10}} \end{aligned}$$

(e) Rationalize the denominator: $\dfrac{3}{\sqrt{7}}$

Solution:

$$\begin{aligned}\frac{3}{\sqrt{7}} &= \frac{3}{\sqrt{7}}\cdot\frac{\sqrt{7}}{\sqrt{7}}\\ &= \boxed{\frac{3\sqrt{7}}{7}}\end{aligned}$$

(f) Add/subtract as indicated (give answer in $a+bi$ form): $i^2+(-7+3i)-\left(-2+\frac{1}{2}i\right)$

Solution:

$$\begin{aligned}i^2+(-7+3i)-\left(-2+\frac{1}{2}i\right) &= -1-7+3i+2-\frac{1}{2}i\\ &= \boxed{-6+\frac{5}{2}i}\end{aligned}$$

2. The following are unrelated: (20 pts)

(a) Simplify: $\sqrt{36x^2y^4}$

Solution: $\sqrt{36x^2y^4} = \boxed{6|x|y^2}$

(b) Simplify: $(2x-1)^2+x^5+4x-6-x^2x^3$.

Solution:

$$\begin{aligned}(2x-1)^2+x^5+4x-6-x^2x^3 &= 4x^2-2x-2x+1+x^5+4x-6-x^2x^3\\ &= 4x^2-4x+1+x^5+4x-6-x^5\\ &= \boxed{4x^2-5}\end{aligned}$$

(c) Multiply: $\left((x+1)^{1/2}+x^{1/2}\right)\left((x+1)^{1/2}+x^{1/2}\right)$

Solution:

$$\begin{aligned}\left((x+1)^{1/2}+x^{1/2}\right)\left((x+1)^{1/2}+x^{1/2}\right) &= (x+1)^{1/2+1/2}+(x+1)^{1/2}x^{1/2}+(x+1)^{1/2}x^{1/2}+x^{1/2+1/2}\\ &= (x+1)+2(x+1)^{1/2}x^{1/2}+x\\ &= \boxed{2x+1+2\sqrt{x(x+1)}}\text{ or }\boxed{2x+1+2x^{1/2}(x+1)^{1/2}}\end{aligned}$$

(d) Simplify (Give your answer without negative exponents): $\left(2x^{-2}\right)^3\dfrac{xy^2}{x^2y^{-3}}$

Solution:

$$\begin{aligned}\left(2x^{-2}\right)^3\frac{xy^2}{x^2y^{-3}} &= 8x^{-6}\cdot\frac{xy^2y^3}{x^2}\\ &= \frac{8xy^2y^3}{x^2x^6}\\ &= \frac{8xy^5}{x^8}\\ &= \boxed{\frac{8y^5}{x^7}}\end{aligned}$$

(e) Is $x = 2$ a solution of $\sqrt{x+2}(x-3)^9(x-1)^7(3) = 6$? Make sure to justify your answer, an answer without work will receive no credit. (4 pts)

Solution: Plug in $x = 2$:

$$\begin{aligned}
&\sqrt{2+2}(2-3)^9(2-1)^7(3) \\
&= \sqrt{4}(-1)^9(1)^7(3) \\
&= 2(-1)(3) \\
&= -6
\end{aligned}$$

Which does not equal 6 so $x = 2$ is not a solution to the equation.

3. The following are unrelated: (17 pts)

(a) Factor completely (If not factorable write NF): $y^2 - 16$

Solution:

$y^2 - 16 = \boxed{(y-4)(y+4)}$ (Difference of Squares)

(b) Factor completely (If not factorable write NF): $x^3 - 2x^2 + 4x - 8$

Solution:

$$\begin{aligned}
x^3 - 2x^2 + 4x - 8 &= x^2(x-2) + 4(x-2) \\
&= \boxed{(x-2)(x^2+4)}
\end{aligned}$$

(c) Find the domain of the expression: $\dfrac{x^2-16}{x(x+4)}$

Solution:

The only restriction on the domain we have is when the denominator, $x(x+4)$, is zero. We see the denominator is zero when $x = 0$ and $x = -4$. Thus, $x = 0$ and $x = -4$ cannot be in the domain of the expression. Therefore, the domain is $\boxed{(-\infty, -4) \cup (-4, 0) \cup (0, \infty)}$.

(d) Simplify the complex fraction: $\dfrac{-\frac{3}{x^2} + \frac{2}{x}}{2 - \frac{1}{x-1}}$

Solution:

$$\begin{aligned}
\frac{-\frac{3}{x^2} + \frac{2}{x}}{2 - \frac{1}{x-1}} &= \frac{-\frac{3}{x^2} + \frac{2x}{x^2}}{\frac{2(x-1)}{x-1} - \frac{1}{x-1}} \\
&= \frac{\frac{-3+2x}{x^2}}{\frac{2(x-1)-1}{x-1}} \\
&= \frac{\frac{2x-3}{x^2}}{\frac{2x-2-1}{x-1}} \\
&= \frac{2x-3}{x^2} \div \frac{2x-3}{x-1} \\
&= \frac{2x-3}{x^2} \cdot \frac{x-1}{2x-3} \\
&= \boxed{\frac{x-1}{x^2}}
\end{aligned}$$

(e) Divide and simplify: $\frac{\frac{38}{x^2-x}}{\frac{16}{x^3-x^2}}$

Solution:

$$\begin{aligned}
\frac{\frac{38}{x^2-x}}{\frac{16}{x^3-x^2}} &= \frac{38}{x^2-x} \cdot \frac{x^3-x^2}{16} \\
&= \frac{38}{x(x-1)} \cdot \frac{x^2(x-1)}{16} \\
&= \frac{38x^2(x-1)}{16x(x-1)} \\
&= \boxed{\frac{19x}{8}}
\end{aligned}$$

4. Solve each of the following equations: (15 pts)

(a) $x^2 + 12x = -20$

Solution:

$$\begin{aligned}
x^2 + 12x &= -20 \\
x^2 + 12x + 20 &= 0 \\
(x+10)(x+2) &= 0
\end{aligned}$$

By the multiplicative property of zero this results in $x+10 = 0$ and $x+2 = 0$. These two equations result in $\boxed{x = -10}$ and $\boxed{x = -2}$.

(b) $\sqrt{2} + 2x = 7 + x$

Solution:

$$\begin{aligned}
\sqrt{2} + 2x &= 7 + x \\
x &= \boxed{7 - \sqrt{2}}
\end{aligned}$$

(c) $x + 4 = 1 + \sqrt{x+5}$

Solution:

$$\begin{aligned}
x + 4 &= 1 + \sqrt{x+5} \\
x + 3 &= \sqrt{x+5} \\
(x+3)^2 &= \left(\sqrt{x+5}\right)^2 \\
x^2 + 6x + 9 &= x + 5 \\
x^2 + 5x + 4 &= 0 \\
(x+4)(x+1) &= 0
\end{aligned}$$

Resulting in potential solutions of $x = -1$ and $x = -4$. Checking these values in the original equation reveals that only $\boxed{x = -1}$ is in fact a solution.

5. Solve each of the following equations: (10 pts)

   (a) $\dfrac{x}{3x^2} + \dfrac{1}{6x} = \dfrac{2-x}{6x^2}$

   Solution:

$$\begin{aligned}
\frac{x}{3x^2} + \frac{1}{6x} &= \frac{2-x}{6x^2} \\
\frac{2x}{6x^2} + \frac{x}{6x^2} &= \frac{2-x}{6x^2} \\
\frac{3x}{6x^2} &= \frac{2-x}{6x^2} \\
3x &= 2-x \\
4x &= 2 \\
x &= \frac{1}{2}
\end{aligned}$$

   So the answer is $\boxed{x = \frac{1}{2}}$.

   (b) Solve for M: $\quad 3M - 2PM = 4 + M$

   Solution:

$$\begin{aligned}
3M - 2PM &= 4 + M \\
2M - 2PM &= 4 \\
2M(1-P) &= 4 \\
M &= \frac{4}{2(1-P)} \\
M &= \frac{2}{1-P}
\end{aligned}$$

   So the answer is $\boxed{M = \frac{2}{1-P}}$.

6. Solve the following inequalities. Justify your answers by using a number line or sign chart. Answers without full justification will not receive full credit. Express all answers in interval notation. (20 pts)

   (a) $1 - 4x \leq 3$

   Solution:

$$\begin{aligned}
1 - 4x &\leq 3 \\
-4x &\leq 2 \\
x &\geq -\frac{1}{2}
\end{aligned}$$

   In interval notation: $\boxed{\left[-\frac{1}{2}, \infty\right)}$

   (b) $(x-1)^2(x+3) \geq 0$

   Solution:

   We see that the left hand side is in factored form. Solving: $(x-1)^2(x+3) = 0$ we find our key values: $x = 1$ and $x = -3$. Plotting these on a number line and testing values we get a solution of $\boxed{[-3, \infty)}$.

(c) $|x-5|<0.1$

Solution:

$|x-5| < 0.1$ is solved when $-0.1 < x-5 < 0.1$ or $4.9 < x < 5.1$. In interval notation: $\boxed{(4.9, 5.1)}$

(d) $\dfrac{-3x}{x+3} \geq 0$

Solution:

We can find the key values by setting the numerator equal to zero resulting in $-3x = 0$ and $x = 0$. We also set the denominator equal to zero resulting in $x+3 = 0$ and $x = -3$. Putting these values on a number line and choosing test values we get the answer: $\boxed{(-3, 0]}$. Note that we need to exclude $x = -3$ since this value does not solve the original inequality.