APPM 1235 **Final Exam** **Fall 2021**

INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources) are not permitted. **Give all answers in exact form.**

Potentially useful formulas:

1. $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{\cos(2\theta)+1}{2}$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND THE NEXT PAGE AND USE (FRONT AND BACK) OF BOTH AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE

iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE. JUST BEFORE YOU UPLOAD TO GRADESCOPE WRITE DOWN YOUR UPLOAD TIME ON THE FIRST EXAM PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND LINE UP AT THE BACK OF THE ROOM. A PROCTOR WILL INDICATE WHEN IT'S YOUR TURN TO EXIT THE ROOM AND UPLOAD TO GRADESCOPE.

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1. Simplify each of the following. Leave answers without negative exponents. (18 pts)

   (a) $(2a-1)(a-1)-a(a+2)$

   (b) $\left(\dfrac{9q^2p^{-3}}{3^{-1}q^{-4}p^0}\right)(qp)^{-1}$

   (c) $\sqrt{\sqrt[3]{(x^3y^6)^4}}$

   (d) $\left(x^{1/2}-y^{1/3}\right)\left(x^{1/2}+y^{1/3}\right)$

   (e) $\dfrac{x+\frac{1}{x^2}}{1+\frac{1}{x^3}}$

   (f) $(e^x-y)^2-ye^x-\ln\left(ye^{y^2}\right)$

2. Solve the following equations for x: (25 pts)

(a) $(x-2)(x+3)=1$

(b) $\sqrt{-3x+6}+x=-4$

(c) $\dfrac{1}{2x-1}-\dfrac{\sqrt{2}}{2x+1}=-\dfrac{3}{4x^2-1}$

(d) $3^{3x-7} = 9^{x-1}$

(e) $-2\log(2) = \log(3x+1)$

3. For $g(x) = \dfrac{3x - 1}{x^6 - 9x^4}$ answer the following (12 pts):

(a) Find the domain of $g(x)$. Give your answer in interval notation.

(b) Find the x, y-coordinates for any hole(s). If there are none write NONE.

(c) Find any horizontal or slant asymptotes. If there are none write NONE.

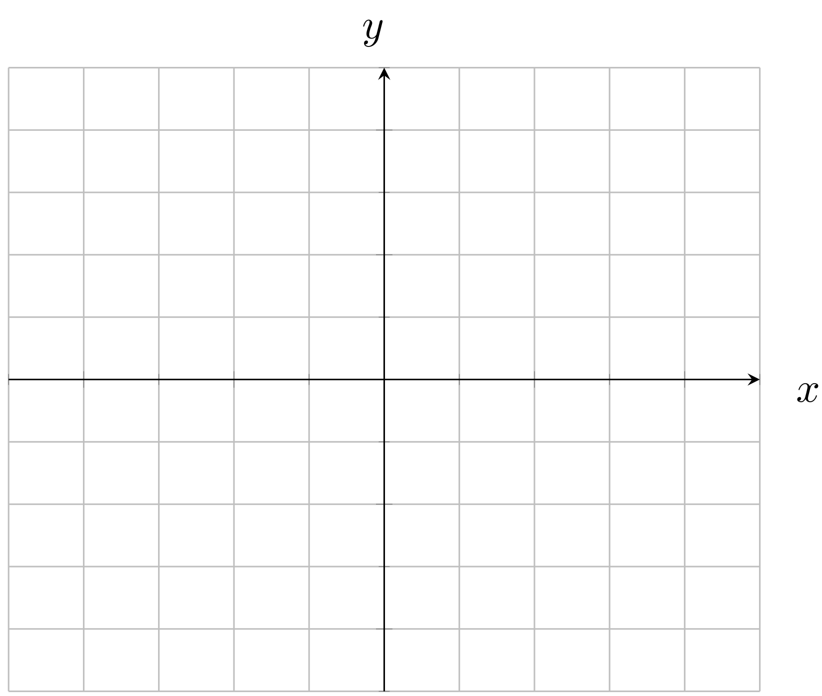
(d) Find any vertical asymptotes. If there are none write NONE.

4. Consider the function $P(x) = -x^4 + x^3 + 6x^2$. Answer the following: (11 pts)

(a) Find all x and y-intercepts.

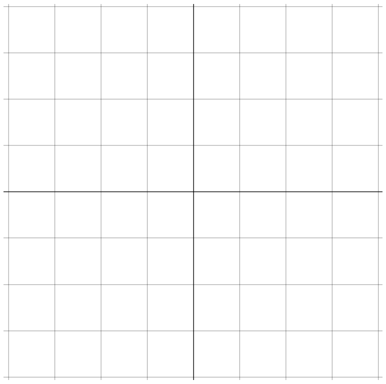
(b) Identify the end behavior (either using arrow notation or depicting on a graph).

(c) Sketch the graph of $P(x)$ be sure to label all x and y-intercepts.

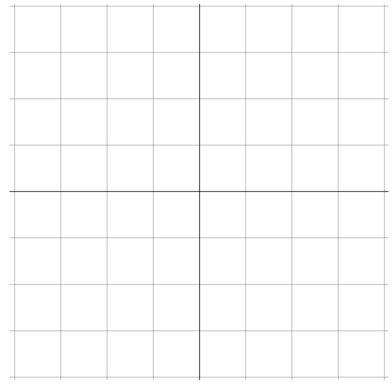


5. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (13 pts)

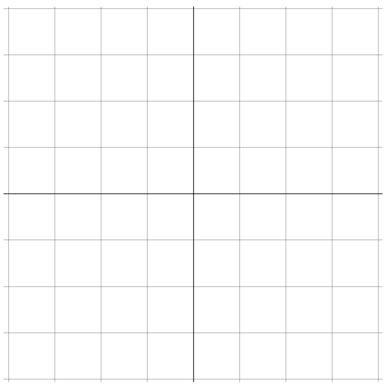
(a) $f(x) = \sqrt{-x} - 1$.



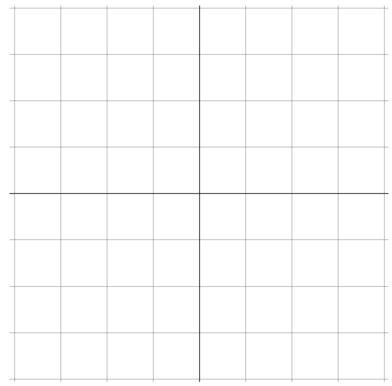
(c) $q(x) = \begin{cases} e^x + 1 & \text{if} \quad x \leq 0 \\ -2x + 1 & \text{if} \quad x > 0 \end{cases}$



(b) $h(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



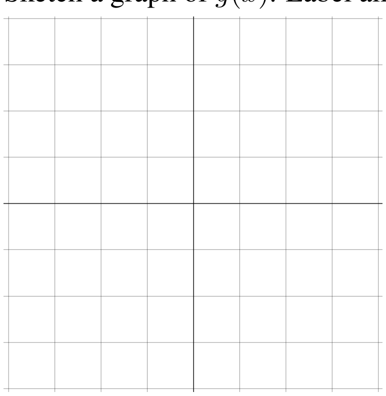
(d) $k(x) = \tan^{-1}(x)$



6. Given $f(x) = \sqrt{x}$ and $g(x) = \log(x)$ answer the following. (8 pts)

(a) Find $(f + g)(1)$.

(b) Sketch a graph of $g(x)$. Label all asymptote(s) and intercept(s).



(c) Find $(f \circ g)(x)$ and find the domain.

7. Find the exact value: (14 pts)

(a) $\cos\left(\frac{5\pi}{6}\right)$

(b) $\csc\left(-\frac{4\pi}{3}\right)$

(c) $\tan^{-1}\left(\sqrt{3}\right)$

(d) $\arccos(1)$

(e) $\cos\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

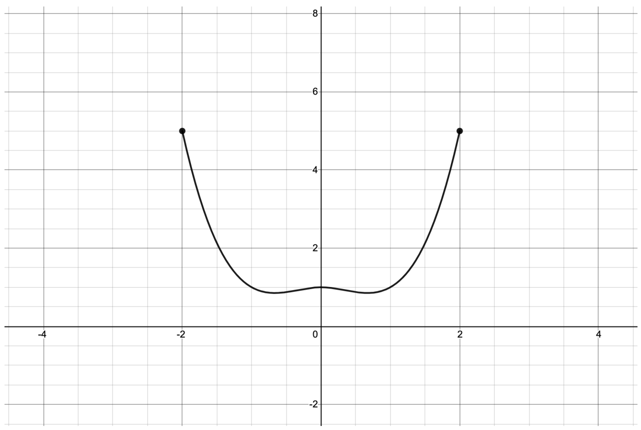
(f) $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

8. For a specific angle θ suppose we know that $\cos\theta < 0$ and θ lies in the interval $[0, \pi]$. What quadrant does θ lie in? (4 pts)

9. Verify the identity: $\frac{\csc^2\theta}{1+\tan^2\theta} = \cot^2\theta$. (6 pts)

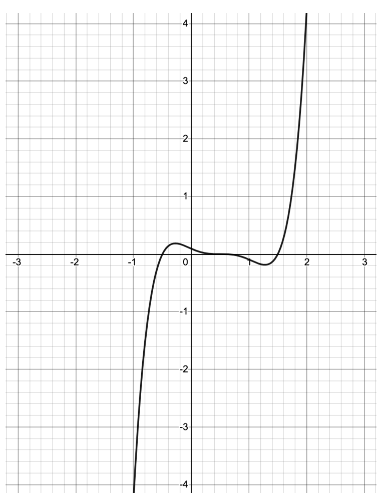
10. The following questions are unrelated. (9 pts)

(a) i. For the graph of $g(x)$ below, with domain $[-2, 2]$, is this the graph of an odd, even, or neither function? No justification is needed.



ii. Does the function $g^{-1}(x)$ exist? Give a brief explanation of why or why not.

(b) For the graph of $h(x)$ below is this the graph of an odd, even, or neither function? No justification is needed.



(c) Is $f(x) = \sin x + \cos x$ odd, even, or neither? As usual justify answer for credit.

11. Find all solutions to the following equations: (10 pts)

(a) $\cos\theta + 2\sin\theta\cos\theta = 0$

(b) $\cos\left(\dfrac{\theta}{3}\right) = \dfrac{1}{2}$

12. Find the exact value for each: (10 pts)

(a) $\cos^2(22.5^\circ) - \sin^2(22.5^\circ)$

(b) $\sin\left(-\frac{\pi}{8}\right)$

13. For $f(x) = 3\sin\left(x - \frac{\pi}{3}\right)$ (10 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $f(x)$. Label at least five x-values on the x-axis and amplitude values on the y-axis to receive full credit.