INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources) are not permitted. **Give all answers in exact form.**

Potentially useful formulas:

Let u and w denote positive real numbers, then:

(a) $\log_b(uv) = \log_b(u) + \log_b(v)$

(b) $\log_b\left(\frac{u}{v}\right) = \log_b(u) - \log_b(v)$

(c) $\log_b(u^c) = c\log_b(u)$ where c is any real number.

(d) $\log_b(u) = \frac{\log_a(u)}{\log_a(b)}$ for $a > 0$, $a \neq 1$.

(e) $A = \frac{1}{2}r^2\theta$

(f) $S = r\theta$

NOTE: YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

iii. WRITE YOUR NAME ON THE NEXT PAGE. JUST BEFORE YOU UPLOAD TO GRADESCOPE WRITE DOWN YOUR UPLOAD TIME ON THE NEXT PAGE.

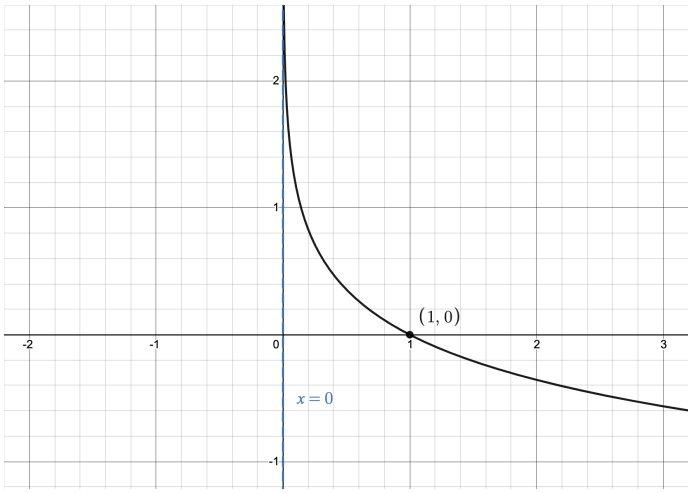
iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND LINE UP AT THE BACK OF THE ROOM. A PROCTOR WILL INDICATE WHEN IT'S YOUR TURN TO EXIT THE ROOM AND UPLOAD TO GRADESCOPE.

Name: _____ Upload time: _____

1. Sketch the following graphs: Be sure to label any asymptotes and intercepts for each graph.

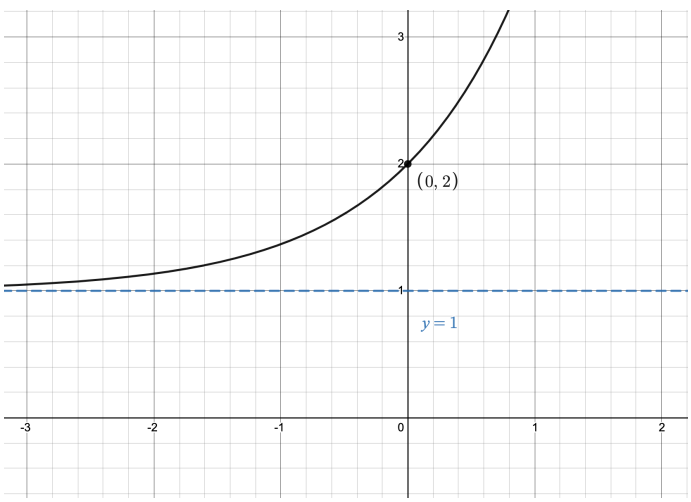
   (a) $f(x) = -\log_7(x)$ (4 pts)

   Solution



   (b) $g(x) = e^x + 1$ (4 pts)

   Solution



   (c) For $f(x)$ given in part (a) find $f\left(7^{2x}\right)$. (3 pts)

   Solution

   $f\left(7^{2x}\right) = -\log_7\left(7^{2x}\right) = -2x.$

2. (a) Simplify (rewrite without logs): $\ln(1) - e^{2\ln(5)} + \log_5(125)$ (3 pts)

Solution

$\ln(1) - e^{2\ln 5} + \log_5(125) = 0 - e^{\ln(5^2)} + 3 = -25 + 3 = -22.$

(b) Rewrite as a single logarithm without negative exponents: $-4\log_3(x) + \log_3(y) - 7\log_3(z)$ (4 pts)

Solution

$$\begin{aligned}
&-4\log_3(x) + \log_3(y) - 7\log_3(z) && (1)\\
&= \log_3\left(x^{-4}\right) + \log_3(y) - \log_3\left(z^7\right) && (2)\\
&= \log_3\left(x^{-4}y\right) - \log_3\left(z^7\right) && (3)\\
&= \log_3\left(\frac{x^{-4}y}{z^7}\right) && (4)\\
&= \log_3\left(\frac{y}{x^4z^7}\right) && (5)
\end{aligned}$$

(c) Rewrite as a sum/difference of logarithms without any exponents: $\log\left(\frac{\sqrt{xy}}{z}\right)$ (4 pts)

Solution

$$\begin{aligned}
&\log\left(\frac{\sqrt{xy}}{z}\right) && (6)\\
&= \log\left(\sqrt{xy}\right) - \log(z) && (7)\\
&= \log\left((xy)^{1/2}\right) - \log(z) && (8)\\
&= \frac{1}{2}\log(xy) - \log(z) && (9)\\
&= \frac{1}{2}\left(\log(x) + \log(y)\right) - \log(z) && (10)\\
&= \frac{1}{2}\log(x) + \frac{1}{2}\log(y) - \log(z) && (11)
\end{aligned}$$

3. Solve the following equations for x. If there are no solutions write "no solutions" (be sure to justify answer for full credit).

(a) $\log_x(32) = 2$ (4 pts)

Solution

$$\begin{aligned}
\log_x(32) &= 2 && (12)\\
x^2 &= 32 && (13)\\
x &= \sqrt{32} && (14)\\
x &= 4\sqrt{2} && (15)
\end{aligned}$$

(b) $e^{x^2-1} = e^{6(x+1)}$ (4 pts)

Solution

$$
\begin{aligned}
e^{x^2-1} &= e^{6(x+1)} && (16)\\
x^2 - 1 &= 6(x+1) && (17)\\
x^2 - 1 &= 6x + 6 && (18)\\
x^2 - 6x - 7 &= 0 && (19)\\
(x-7)(x+1) &= 0 && (20)
\end{aligned}
$$

Resulting in answers: $x = -1, 7$.

(c) $2^{x+1} = 4^{x-1}$ (4 pts)

Solution

$$
\begin{aligned}
2^{x+1} &= 4^{x-1} && (21)\\
\log_2\left(2^{x+1}\right) &= \log_2\left(4^{x-1}\right) && (22)\\
\log_2\left(2^{x+1}\right) &= (x-1)\log_2(4) && (23)\\
x+1 &= (x-1)2 && (24)\\
x+1 &= 2x-2 && (25)\\
x &= 3 && (26)
\end{aligned}
$$

(d) $2 = \log_3(2x-21) - \log_3(x)$ (4 pts)

Solution

$$
\begin{aligned}
2 &= \log_3(2x-21) - \log_3(x) && (27)\\
2 &= \log_3\left(\frac{2x-21}{x}\right) && (28)\\
3^2 &= \frac{2x-21}{x} && (29)\\
9x &= 2x-21 && (30)\\
7x &= -21 && (31)\\
x &= -3 && (32)
\end{aligned}
$$

Checking if $x = -3$ is a solution we find that it does not solve the original equation so the original equation has no solutions.

(e) $8 + 3x = x\ln(8) - 2$ (4 pts)

Solution

$$
\begin{aligned}
8 + 3x &= x\ln(8) - 2 && (33)\\
3x - x\ln(8) &= -10 && (34)\\
x(3 - \ln(8)) &= -10 && (35)\\
x &= -\frac{10}{3 - \ln(8)} && (36)
\end{aligned}
$$

4. The velocity of a sky diver t seconds after jumping is modeled by $v(t) = 70\left(1 - e^{-0.3t}\right)$. After how many seconds is the velocity 7 ft/s? (Give your answer as an exact value) (4 pts)

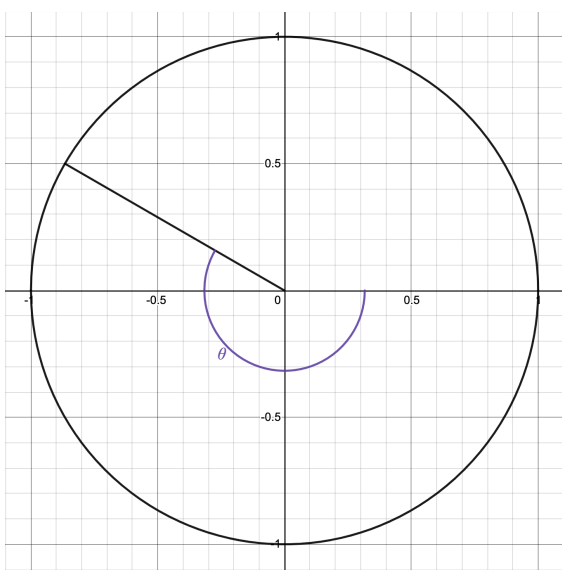
Solution

$$
\begin{aligned}
v(t) &= 70\left(1 - e^{-0.3t}\right) && (37)\\
7 &= 70\left(1 - e^{-0.3t}\right) && (38)\\
\frac{1}{10} &= 1 - e^{-0.3t} && (39)\\
-\frac{9}{10} &= -e^{-0.3t} && (40)\\
\frac{9}{10} &= e^{-0.3t} && (41)\\
\ln\left(\frac{9}{10}\right) &= -0.3t && (42)\\
t &= -\frac{10}{3}\ln\left(\frac{9}{10}\right) && (43)
\end{aligned}
$$

5. Sketch each angle in standard position on the unit circle.

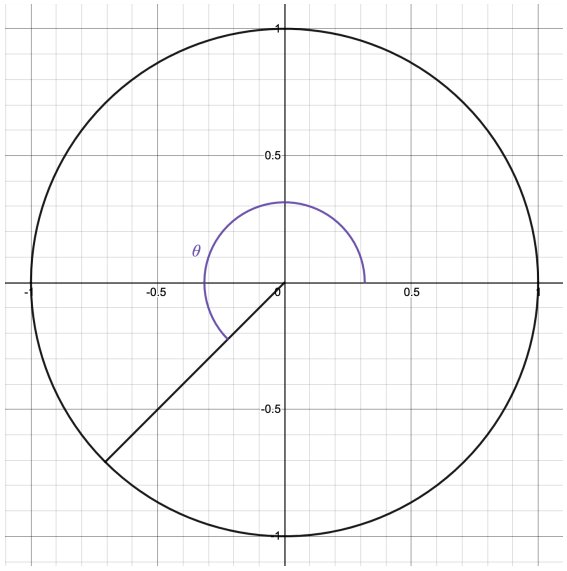
(a) $-\frac{7\pi}{6}$ (3 pts)

Solution



(b) $\dfrac{5\pi}{4}$ (3 pts)

Solution



6. The point $(-2, -3)$ is on the terminal side of an angle, θ, in standard position. Determine the exact values of the following.

(a) $\cos\theta$ (4 pts)

Solution

$$r = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13} \text{ so } \cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}}.$$

(b) $\sec\theta$ (3 pts)

Solution

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{2}{\sqrt{13}}} = -\frac{\sqrt{13}}{2}.$$

(c) $\tan\theta$ (4 pts)

Solution

$$\tan\theta = \frac{y}{x} = \frac{-3}{-2} = \frac{3}{2}.$$

7. The length of the arc of the sector of a circle with a central angle of 20° is 4 m. Find the radius of the circle. (Give your answer in exact form.) (4 pts)

Solution

We can use $s = r\theta$ but this formula only works if θ is in radians. $\theta = (20^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{9}$. So $4 = r\left(\frac{\pi}{9}\right)$ and $r = \frac{36}{\pi}$ m.

8. Simplify the following:

   (a) $2\cos^2(49.8^\circ) + 2\sin^2(49.8^\circ)$ (4 pts)

   Solution

   $2\cos^2(49.8^\circ) + 2\sin^2(49.8^\circ) = 2\left(\cos^2(49.8^\circ) + \sin^2(49.8^\circ)\right) = 2(1) = 2.$

   (b) For a particular angle θ in standard position suppose we know $\tan\theta < 0$ and $\cos\theta > 0$. What quadrant is θ in? (4 pts)

   Solution

   $\tan\theta < 0$ in quadrants II and IV. $\cos\theta > 0$ in quadrants I and IV. So θ is in quadrant IV.

9. Find the following. If a value does not exist write DNE.

   (a) $\tan(0^\circ)$ (4 pts)

   Solution

   $\tan(0^\circ) = 0.$

   (b) $\sin\left(\frac{7\pi}{6}\right)$ (4 pts)

   Solution

   $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

   (c) $\cos\left(\frac{2\pi}{3}\right)$ (4 pts)

   Solution

   $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

   (d) $\cot(-45^\circ)$ (4 pts)

   Solution

   $\cot(-45^\circ) = -1$

(e) $\sec\left(\frac{\pi}{6}\right)$ (4 pts)

Solution

$$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

10. A mountain stands in the distance 10,040 feet tall. You measure the angle from your feet to the top of the mountain to be $60°$. How far away is the base of the mountain? Give your answer in exact form. (5 pts)

Solution

Let d be your distance from the base of the mountain. We can write: $\tan(60°) = \frac{10040}{d}$. Since $\tan(60°) = \frac{\sin(60°)}{\cos(60°)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ then solving for d we get: $d = \frac{10040}{\tan(60°)} = \frac{10040}{\sqrt{3}}$ ft.