INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted while taking the exam. The exam is worth 100 points.

Potentially useful formulas:

(i) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(ii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(iii) Equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

WRITE YOUR NAME ON THE NEXT PAGE. JUST BEFORE YOU UPLOAD TO GRADESCOPE WRITE DOWN YOUR UPLOAD TIME ON THE NEXT PAGE.

ONCE A PROCTOR HAS INDICATED YOU CAN START YOU MAY TEAR OFF THIS COVER PAGE AND USE IT AS SCRATCH PAPER.

1. Given the equation of a circle: $(x + 3)^2 + (y - 2)^2 = 15$ answer the following: (8 pts)

    (a) Find the coordinates of the center of the circle and state the radius.

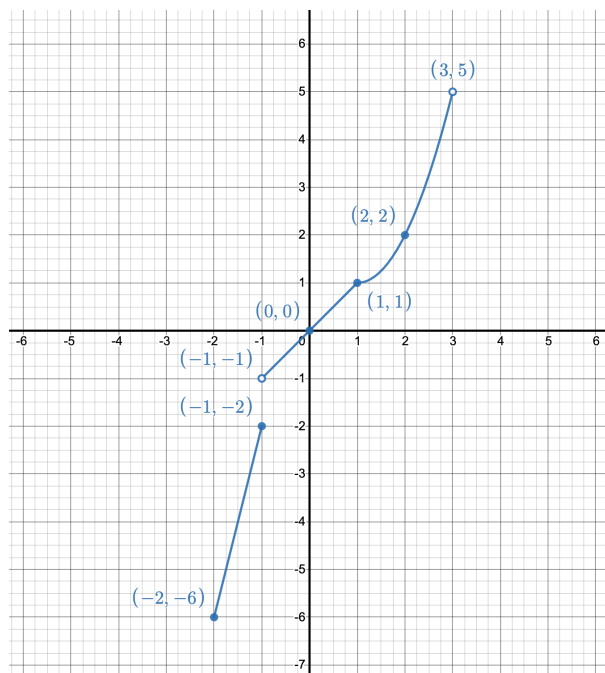   (b) Find the equation for the upper half of the circle.

2. Find the domain of the following functions. Express your answers in interval notation. (15 pts)

(a) $f(x) = \dfrac{x^2 - 2x + 1}{x^2 + 2x - 15}$

(b) $g(x) = \dfrac{x}{\sqrt{2x^2 - 4x}}$

(c) $h(x) = 2x^3 - 4x^2 + 6$

3. Answer the following for the given graph of $f(x)$. (14 pts)

(a) Find the domain of $f(x)$. Express your answer in interval notation.

(b) Find the range of $f(x)$. Express your answer in interval notation.

(c) Find $f(0)$.

(d) Find $(f + f)(2)$.

(e) Find $(f \circ f)(-1)$.

(f) $f(x)$ is one-to-one, give a brief sentence to explain why it is one-to-one.

(g) Find the range of $f^{-1}(x)$. Express your answer in interval notation.

4. Sketch the shape of the graph of each of the following on the given set of axes. Label x-intercepts if any: (19 pts)
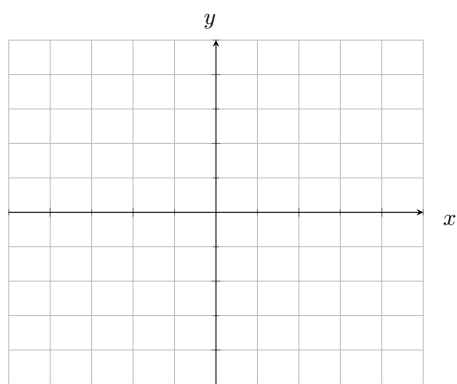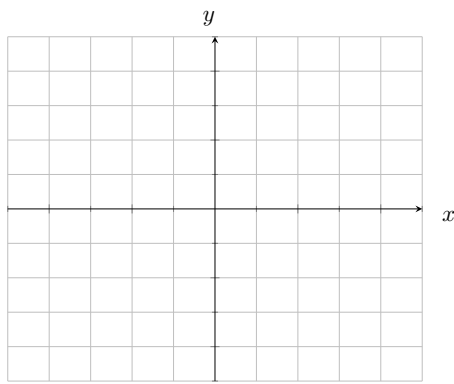
(a) $f(x) = x^3 - 1$
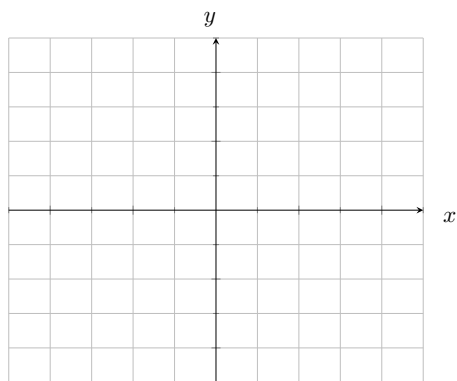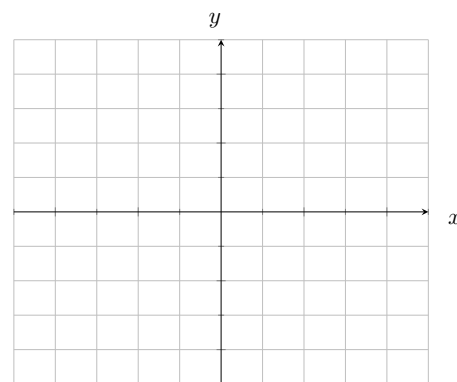
(d) $g(x) = \sqrt[3]{x+2}$
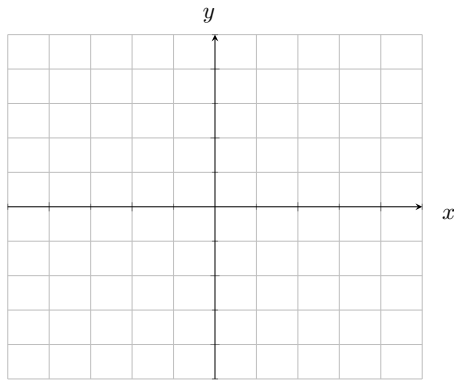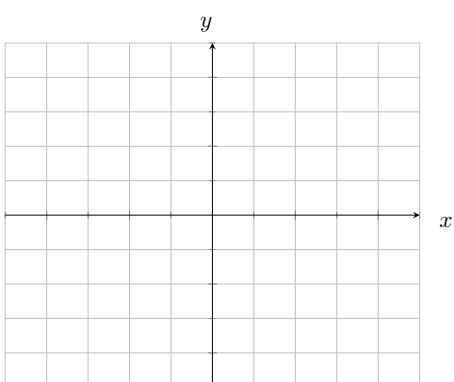
(b) $k(x) = \dfrac{1}{x}$

(e) $m(x) = -|x|$

(c) $(x-1)^2 + (y+2)^2 = 4$

(f) $q(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$

5. For $g(x) = x^3 - 2$ and $h(x) = \dfrac{1}{\sqrt[3]{x}}$ answer the following: (16 pts)

(a) Is $g(x)$ odd, even, or neither? Justify your answer to receive credit.

(b) Find $h\left(\sqrt{x}\right)$ and rewrite your answer with rational exponent(s). Simplify your answer as usual.

(c) Find $(g \circ h)(x)$ and state its domain in interval notation.

(d) Find $g^{-1}(x)$ (you may assume the inverse function exists).

6. Consider the given graph of a polynomial $P(x)$ with domain $(-\infty, \infty)$. Write down a degree four polynomial, in factored form, whose graph is consistent with that of $P(x)$: (5 pts)
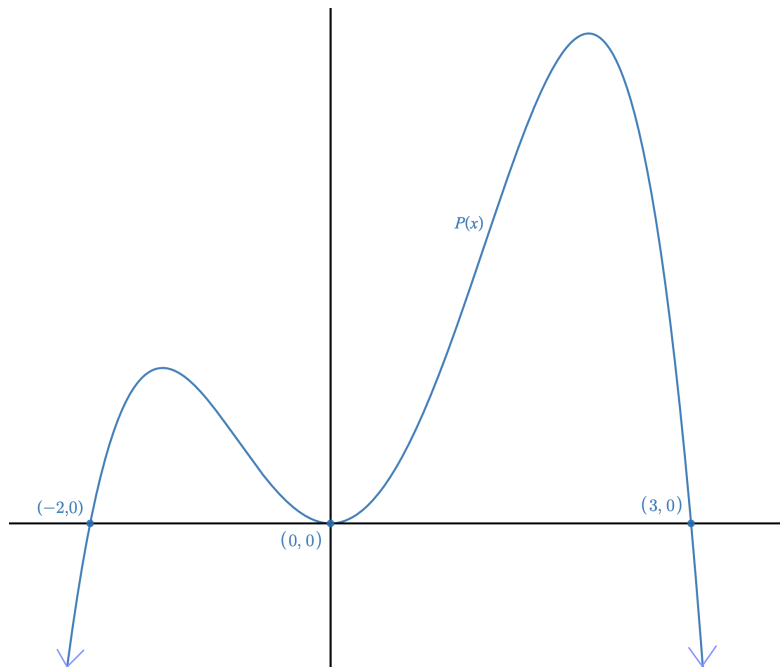
7. Sketch the shape of the graph of a rational function, $R(x)$, that satisfies **all** of the information. **Label** all intercepts and asymptotes on the graph. (6 pts)

   i. The graph has a horizontal asymptote: $y = -2$.

  ii. The graph crosses at x-intercept $(2, 0)$.

 iii. The graph bounces (touches but does not cross) at x-intercept $(-1, 0)$

 iv. The graph has no other x-intercepts and no holes.

  v. The graph has y-intercept $\left(0, \dfrac{1}{2}\right)$.

 vi. The graph has a vertical asymptote: $x = -2$
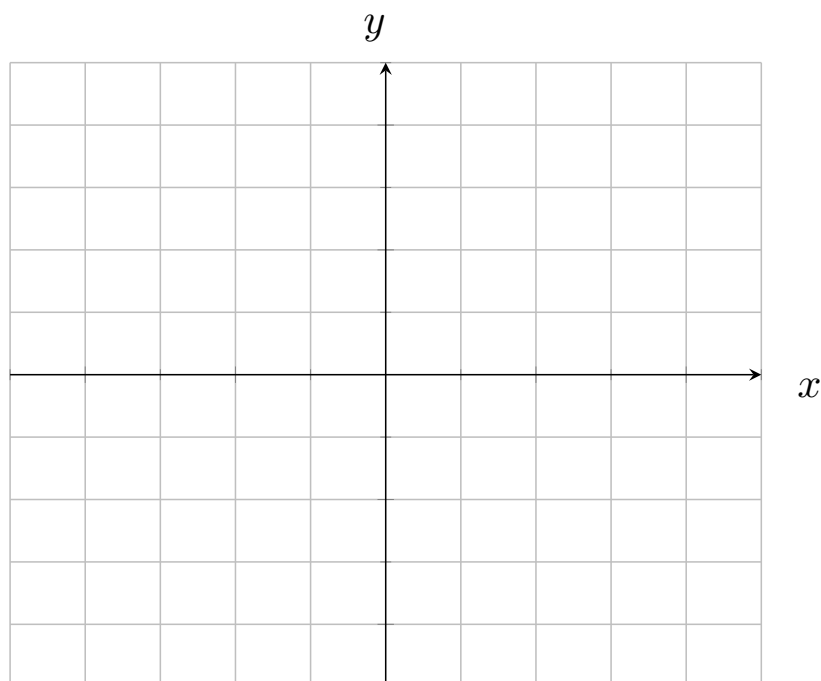
8. For $R(x) = \dfrac{3x^3 - 6x^2 - 9x}{x^2 - 9}$ (12 pts)

    (a) Find the location (x, y -coordinates) of any hole(s). If there are none state NONE.

    (b) Find any horizontal or slant asymptote(s). If there are none state NONE.

    (c) Find any vertical asymptote(s). If there are none state NONE.

9. The surface of a swimming pool is being designed to have the shape of a semi-circle adjoined to a rectangle with total perimeter 120 ft (see diagram). The diameter of the circle is half the length of one side of the rectangle (as depicted on diagram). The length of one side of the rectangle is y. Write down the area of the pool as a function of x. **You may leave your final answer in an unsimplified form.** (5 pts)

$\frac{x}{2}$

$\frac{x}{2}$

y

x