INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted. The exam is worth 100 points. **Assume all questions have answers in the real numbers unless the instructions specify complex numbers.**

Potentially useful formulas:

(i) $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ (ii) $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

NAME: ____

1. The following are unrelated:

   (a) Simplify: $2 - 3i - (-1 + i^2)$.

   Solution:

   $2 - 3i - (-1+i^2) = 2 - 3i - (-1+-1) = 2 - 3i - (-2) = 2 - 3i + 2 = 4 - 3i.$

   (b) Multiply: $\left(x^{1/2} + y^{1/2}\right)\left(x^{1/2} - y^{1/2}\right)$

   Solution:

   $\left(x^{1/2} + y^{1/2}\right)\left(x^{1/2} - y^{1/2}\right) = \left(x^{1/2}\right)^2 + x^{1/2}y^{1/2} - y^{1/2}x^{1/2} - \left(y^{1/2}\right)^2 = x - y.$

   (c) Simplify. Leave answer in factored form: $\dfrac{2^{-3}(x+1)^{-3}}{4^{-1/2}(x+1)^5}$

   Solution:

   $$\frac{2^{-3}(x+1)^{-3}}{4^{-1/2}(x+1)^5} = \frac{4^{1/2}}{2^3(x+1)^8} = \frac{2}{2^3(x+1)^8} = \frac{1}{2^2(x+1)^8} = \frac{1}{4(x+1)^8}.$$

   (d) Factor completely (If not factorable write NF): $y^2 - 4x^2$

   Solution:

   $y^2 - 4x^2 = (y-2x)(y+2x).$

   (e) Combine into a single fraction: $\dfrac{2}{x^2+x} - \dfrac{1}{x^2+4x+3}$

   Solution:

   $$\frac{2}{x^2+x} - \frac{1}{x^2+4x+3} = \frac{2}{x(x+1)} - \frac{1}{(x+3)(x+1)} = \frac{x+3}{x+3}\cdot\frac{2}{x(x+1)} - \frac{x}{x}\cdot\frac{1}{(x+3)(x+1)} =$$
   $$\frac{2x+6}{x(x+1)(x+3)} - \frac{x}{x(x+1)(x+3)} = \frac{2x+6-x}{x(x+1)(x+3)} = \frac{x+6}{x(x+1)(x+3)}.$$

(f) Simplify the complex fraction: $\dfrac{\frac{1}{x-1} - \frac{1}{2(x-1)}}{3 + \frac{1}{x-1}}$

Solution:

$$\frac{\frac{1}{x-1} - \frac{1}{2(x-1)}}{3 + \frac{1}{x-1}} = \frac{\frac{2}{2} \cdot \frac{1}{x-1} - \frac{1}{2(x-1)}}{\frac{x-1}{x-1} \cdot \frac{3}{1} + \frac{1}{x-1}} = \frac{\frac{2}{2(x-1)} - \frac{1}{2(x-1)}}{\frac{3(x-1)}{x-1} + \frac{1}{x-1}} = \frac{\frac{1}{2(x-1)}}{\frac{3x-2}{x-1}} = \frac{1}{2(x-1)} \cdot \frac{x-1}{3x-2} = \frac{1}{2(3x-2)}.$$

(g) Simplify: $\sqrt{50x^2y^4}$

Solution:

$$\sqrt{50x^2y^4} = \sqrt{25 \cdot 2x^2y^4} = 5|x|y^2\sqrt{2}.$$

(h) Rewrite with rational exponent(s): $\dfrac{\sqrt[3]{x^2} + \sqrt{x}}{x+2}$

Solution:

$$\frac{\sqrt[3]{x^2} + \sqrt{x}}{x+2} = \frac{\left(x^2\right)^{1/3} + x^{1/2}}{x+2} = \frac{x^{2/3} + x^{1/2}}{x+2}.$$

2. For what value of c is the number $x = 4$ a solution of the equation $\sqrt{x} + cx = 2c$?

Solution:

$$\sqrt{x} + cx = 2c \quad (1)$$
$$\sqrt{4} + c(4) = 2c \quad (2)$$
$$2 + 4c = 2c \quad (3)$$
$$2 = -2c \quad (4)$$
$$-1 = c \quad (5)$$

So $c = -1$ is the value such that $x = 4$ is a solution to the equation.

3. Solve each of the following equations:

(a) $x^2 + 3 = 5x$

Solution:

$$x^2 + 3 = 5x \tag{6}$$
$$x^2 - 5x + 3 = 0 \tag{7}$$

Note that $x^2 - 5x + 3$ does not factor. By quadratic equation: $x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)} = \dfrac{5 \pm \sqrt{25 - 12}}{2} = \dfrac{5 \pm \sqrt{13}}{2}$.

(b) $|3x - 1| = 4$

Solution:

$|3x - 1| = 4$ can be rewritten as $3x - 1 = 4$ and $3x - 1 = -4$ solving each we get: $x = \dfrac{5}{3}$ and $x = -1$.

(c) $\dfrac{x + 5}{x} = \dfrac{5 + x}{x^2}$

Solution:

$$\frac{x + 5}{x} = \frac{5 + x}{x^2} \tag{8}$$
$$x^2 \left(\frac{x + 5}{x}\right) = x^2 \left(\frac{5 + x}{x^2}\right) \tag{9}$$
$$x(x + 5) = 5 + x \tag{10}$$
$$x^2 + 5x = 5 + x \tag{11}$$
$$x^2 + 4x - 5 = 0 \tag{12}$$
$$(x - 1)(x + 5) = 0 \tag{13}$$

This results in solutions $x = 1, -5$. Note these are in fact solutions when we check them in the original equation.

(d) Solve for C: $\dfrac{CK - RK}{2} = -R$

Solution:

$$\frac{CK - RK}{2} = -R \tag{14}$$
$$(2)\left(\frac{CK - RK}{2}\right) = -R(2) \tag{15}$$
$$CK - RK = -2R \tag{16}$$
$$CK = -2R + RK \tag{17}$$
$$C = \frac{-2R + RK}{K} \tag{18}$$

(e) Solve for q: $rq = v + 1 - vq$

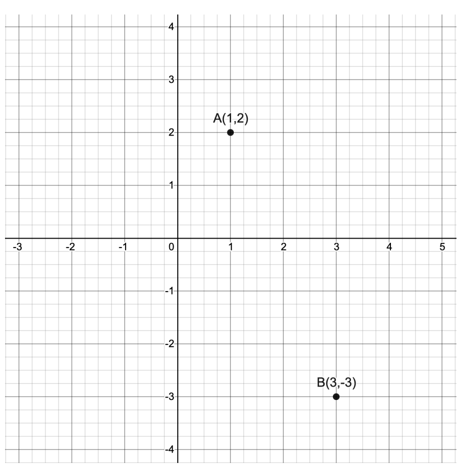
Solution:

$$rq = v + 1 - vq \tag{19}$$
$$rq + vq = v + 1 \tag{20}$$
$$q(r + v) = v + 1 \tag{21}$$
$$q = \frac{v + 1}{r + v} \tag{22}$$

4. Answer each of the following for the points: $A(1, 2)$ and $B(3, -3)$.

(a) Graph the points on a graph with xy-axes. Clearly label points and numerical values on axes.

Solution:



(b) Find the the slope of the line passing through the two points.

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-2}{3-1} = -\frac{5}{2}.$$

(c) Find the equation of the line in $y = mx + b$ (slope-intercept) form.

Solution:

Utilizing $y = mx + b$ we get $y = -\frac{5}{2}x + b$. Plugging in the point $(1, 2)$ we get $2 = -\frac{5}{2}(1) + b$ and solving for b we get $b = \frac{9}{2}$ resulting in equation $y = -\frac{5}{2}x + \frac{9}{2}$.

(d) Find the point that is three-fourths of the way from point B to point A. You may use the fact that $\left(2, -\frac{1}{2}\right)$ is the midpoint between A and B.

Solution:

The point three-fourths of the way from B to A is the midpoint between $\left(2, -\frac{1}{2}\right)$ and $A(1, 2)$. This is found as $\left(\frac{1+2}{2}, \frac{-\frac{1}{2}+2}{2}\right) = \left(\frac{3}{2}, \frac{3}{4}\right)$.

5. Solve the following inequalities. Express all answers in interval notation.

(a) $3 + 3x < x$

Solution:

$$3 + 3x < x \tag{23}$$
$$3 < -2x \tag{24}$$
$$-\frac{3}{2} > x \tag{25}$$

Resulting in solution $\left(-\infty, -\frac{3}{2}\right)$.

(b) $x^3 - 3x^2 > 0$

Solution:

$$x^3 - 3x^2 > 0 \tag{26}$$
$$x^2(x-3) > 0 \tag{27}$$

Setting up a number line and testing values we get solution: $(3, \infty)$.

(c) $|x-2| \leq 5$

Solution:

$|x-2| \leq 5$ can be rewritten $-5 \leq x-2 \leq 5$. Adding 2 to each side of the inequalities we get: $-3 \leq x \leq 7$. In interval notation $[-3, 7]$.

(d) $\dfrac{-3x}{x-5} \leq 0$

Solution:

Setting the numerator equal to zero we get $x = 0$. Setting the denominator equal to zero we get $x = 5$. Putting these values on the real number line and testing values we get the solution: $(-\infty, 0] \cup (5, \infty)$.

6. Find the y-coordinate(s) such that the distance from $(1, 1)$ to $(2, y)$ is 5.

Solution:

Using the distance formula we get:

$$\sqrt{(2-1)^2 + (y-1)^2} = 5 \tag{28}$$
$$(2-1)^2 + (y-1)^2 = 25 \tag{29}$$
$$1 + (y-1)^2 = 25 \tag{30}$$
$$(y-1)^2 = 24 \tag{31}$$
$$y - 1 = \pm\sqrt{24} \tag{32}$$
$$y = 1 \pm 2\sqrt{6} \tag{33}$$

Checking these answers in the original equation we see that they are actual solutions.