INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, and electronic devices are not permitted. The exam is worth 100 points. **Assume all questions have answers in the real numbers unless the instructions specify complex numbers.**

Potentially useful formulas:

(i) \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

(ii) \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]

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NAME: ____________________________

1. The following are unrelated:

(a) Simplify: \[ 2 - 3i - (-1 + i^2) \].

**Solution:**

\[ 2 - 3i - (-1 + i^2) = 2 - 3i - (1 - 1) = 2 - 3i - (-2) = 2 - 3i + 2 = 4 - 3i. \]

(b) Multiply: \( \left( x^{1/2} + y^{1/2} \right) \left( x^{1/2} - y^{1/2} \right) \)

**Solution:**

\[ \left( x^{1/2} + y^{1/2} \right) \left( x^{1/2} - y^{1/2} \right) = \left( x^{1/2} \right)^2 + x^{1/2}y^{1/2} - y^{1/2}x^{1/2} - \left( y^{1/2} \right)^2 = x - y. \]

(c) Simplify. Leave answer in factored form: \( \frac{2^{-3}(x + 1)^{-3}}{4^{-1/2}(x + 1)^5} \)

**Solution:**

\[ \frac{2^{-3}(x + 1)^{-3}}{4^{-1/2}(x + 1)^5} = \frac{4^{1/2}}{2^3(x + 1)^8} = \frac{2}{2^3(x + 1)^8} = \frac{1}{2^2(x + 1)^8} = \frac{1}{4(x + 1)^8}. \]

(d) Factor completely (If not factorable write NF): \( y^2 - 4x^2 \)

**Solution:**

\[ y^2 - 4x^2 = (y - 2x)(y + 2x). \]

(e) Combine into a single fraction: \( \frac{2}{x^2 + x} - \frac{1}{x^2 + 4x + 3} \)

**Solution:**

\[ \frac{2}{x^2 + x} - \frac{1}{x^2 + 4x + 3} = \frac{2}{x(x + 1)} - \frac{1}{2x + 6} = \frac{x + 3}{x(x + 1)(x + 3)} - \frac{2}{x} \cdot \frac{1}{x(x + 1)(x + 3)} = \frac{x + 3}{x(x + 1)(x + 3)} - \frac{2}{x} \cdot \frac{x + 6}{x(x + 1)(x + 3)}. \]
(f) Simplify the complex fraction: \( \frac{\frac{1}{x-1} - \frac{1}{2(x-1)}}{3 + \frac{1}{x-1}} \)

Solution:

\[
\frac{\frac{1}{x-1} - \frac{1}{2(x-1)}}{3 + \frac{1}{x-1}} = \frac{\frac{2}{x-1} \cdot \frac{1}{x-1} - \frac{1}{2(x-1)}}{3 + \frac{1}{x-1}} = \frac{\frac{2}{2(x-1)} - \frac{1}{2(x-1)}}{\frac{3(x-1)}{x-1} + \frac{1}{x-1}} = \frac{\frac{1}{2(x-1)} \cdot \frac{x-1}{3(x-1)}}{3 - 2} = \frac{1}{2(3x - 2)}.
\]

(g) Simplify: \( \sqrt{50x^2y^4} \)

Solution:

\[
\sqrt{50x^2y^4} = \sqrt{25 \cdot 2x^2y^4} = 5|x|y^2\sqrt{2}.
\]

(h) Rewrite with rational exponent(s): \( \frac{\sqrt{x^2} + \sqrt{x}}{x + 2} \)

Solution:

\[
\frac{\sqrt{x^2} + \sqrt{x}}{x + 2} = \frac{(x^2)^{1/3} + x^{1/2}}{x + 2} = \frac{x^{2/3} + x^{1/2}}{x + 2}.
\]

2. For what value of \( c \) is the number \( x = 4 \) a solution of the equation \( \sqrt{x} + cx = 2c \)?

Solution:

\[
\sqrt{x} + cx = 2c \quad (1)
\]

\[
\sqrt{4} + c(4) = 2c \quad (2)
\]

\[
2 + 4c = 2c \quad (3)
\]

\[
2 = -2c \quad (4)
\]

\[
-1 = c \quad (5)
\]

So \( c = -1 \) is the value such that \( x = 4 \) is a solution to the equation.
3. Solve each of the following equations:

(a) \( x^2 + 3 = 5x \)

Solution:

\[
x^2 + 3 = 5x
\]

\[
x^2 - 5x + 3 = 0
\]

Note that \( x^2 - 5x + 3 \) does not factor. By quadratic equation:

\[
x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)} = \frac{5 \pm \sqrt{13}}{2}.
\]

(b) \( |3x - 1| = 4 \)

Solution:

\[
|3x - 1| = 4 \text{ can be rewritten as } 3x - 1 = 4 \text{ and } 3x - 1 = -4 \text{ solving each we get: } x = \frac{5}{3} \text{ and } x = -1.
\]

(c) \( \frac{x + 5}{x} = \frac{5 + x}{x^2} \)

Solution:

\[
\frac{x + 5}{x} = \frac{5 + x}{x^2}
\]

\[
x^2 \left( \frac{x + 5}{x} \right) = x^2 \left( \frac{5 + x}{x^2} \right)
\]

\[
x(x + 5) = 5 + x
\]

\[
x^2 + 5x = 5 + x
\]

\[
x^2 + 4x - 5 = 0
\]

\[
(x - 1)(x + 5) = 0
\]

This results in solutions \( x = 1, -5 \). Note these are in fact solutions when we check them in the original equation.
(d) Solve for $C$: \[ \frac{CK - RK}{2} = -R \]

Solution:

\[ \frac{CK - RK}{2} = -R \]  \hspace{1cm} (14)

\[ \left( \frac{CK - RK}{2} \right) = -R(2) \]  \hspace{1cm} (15)

\[ CK - RK = -2R \]  \hspace{1cm} (16)

\[ CK = -2R + RK \]  \hspace{1cm} (17)

\[ C = \frac{-2R + RK}{K} \]  \hspace{1cm} (18)

(e) Solve for $q$: \[ rq = v + 1 - vq \]

Solution:

\[ rq = v + 1 - vq \]  \hspace{1cm} (19)

\[ rq + vq = v + 1 \]  \hspace{1cm} (20)

\[ q(r + v) = v + 1 \]  \hspace{1cm} (21)

\[ q = \frac{v + 1}{r + v} \]  \hspace{1cm} (22)

4. Answer each of the following for the points: $A(1, 2)$ and $B(3, -3)$.

(a) Graph the points on a graph with $xy$-axes. Clearly label points and numerical values on axes.

Solution:
(b) Find the slope of the line passing through the two points.

Solution:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 2}{3 - 1} = -\frac{5}{2}.
\]

(c) Find the equation of the line in \( y = mx + b \) (slope-intercept) form.

Solution:

Utilizing \( y = mx + b \) we get \( y = -\frac{5}{2}x + b \). Plugging in the point \((1, 2)\) we get \( 2 = -\frac{5}{2}(1) + b \) and solving for \( b \) we get \( b = \frac{9}{2} \) resulting in equation \( y = -\frac{5}{2}x + \frac{9}{2} \).

(d) Find the point that is three-fourths of the way from point \( B \) to point \( A \). You may use the fact that \( \left(2, -\frac{1}{2}\right) \) is the midpoint between \( A \) and \( B \).

Solution:

The point three-fourths of the way from \( B \) to \( A \) is the midpoint between \( \left(2, -\frac{1}{2}\right) \) and \( A(1, 2) \). This is found as \( \left(\frac{1 + 2}{2}, -\frac{1}{2} + 2\right) = \left(\frac{3}{2}, \frac{3}{4}\right) \).

5. Solve the following inequalities. Express all answers in interval notation.

(a) \( 3 + 3x < x \)

Solution:

\[
3 + 3x < x \\
3 < -2x \\
-\frac{3}{2} > x
\]

Resulting in solution \( (-\infty, -\frac{3}{2}) \).

(b) \( x^3 - 3x^2 > 0 \)

Solution:

\[
x^3 - 3x^2 > 0 \\
x^2(x - 3) > 0
\]

Setting up a number line and testing values we get solution: \((3, \infty)\).
(c) \(|x - 2| \leq 5\)

Solution:

\(|x - 2| \leq 5\) can be rewritten \(-5 \leq x - 2 \leq 5\). Adding 2 to each side of the inequalities we get:
\[-3 \leq x \leq 7.\] In interval notation \([-3, 7]\).

(d) \(\frac{-3x}{x - 5} \leq 0\)

Solution:

Setting the numerator equal to zero we get \(x = 0\). Setting the denominator equal to zero we get \(x = 5\). Putting these values on the real number line and testing values we get the solution:
\((-\infty, 0] \cup (5, \infty)\).

6. Find the \(y\)-coordinate(s) such that the distance from \((1, 1)\) to \((2, y)\) is 5.

Solution:

Using the distance formula we get:

\[
\sqrt{(2 - 1)^2 + (y - 1)^2} = 5
\]
\[
(2 - 1)^2 + (y - 1)^2 = 25
\]
\[
1 + (y - 1)^2 = 25
\]
\[
(y - 1)^2 = 24
\]
\[
y - 1 = \pm \sqrt{24}
\]
\[
y = 1 \pm 2\sqrt{6}
\]

Checking these answers in the original equation we see that they are actual solutions.