1. Sketch the graph of: (24 pts)

(a) \( y = (x - 3)^2 \)

Solution:

(b) \( y = \sqrt{x} - 2 \)

Solution:
(c) \( y = x^{1/3} \)

Solution:

![Graph of \( y = x^{1/3} \)](image)

(d) \( y = x^3 \)

Solution:

![Graph of \( y = x^3 \)](image)

(e) \( y = | -x| \)

Solution:

![Graph of \( y = | -x| \)](image)
(f) \( y = -2x^4 \)

Solution:

\[
\begin{align*}
\text{(a)} & \quad (fg)(x) \\
\text{Solution:} & \\
(fg)(x) &= f(x) \cdot g(x) = (x^2 - 4) \left( \frac{1}{x} \right) = \frac{x^2 - 4}{x}. \text{ Domain is: } (-\infty, 0) \cup (0, \infty) \\
\text{(b)} & \quad \left( \frac{f}{g} \right)(x) \\
\text{Solution:} & \\
\left( \frac{f}{g} \right)(x) &= \frac{f(x)}{g(x)} = \frac{x^2 - 4}{1/x} = x^3 - 4x. \text{ Domain is: } (-\infty, 0) \cup (0, \infty) \\
\text{(c)} & \quad (g \circ f)(x) \\
\text{Solution:} & \\
(g \circ f)(x) &= g(f(x)) = \frac{1}{x^2 - 4}. \text{ Domain is: } (-\infty, -2) \cup (-2, 2) \cup (2, \infty) \\
\end{align*}
\]
3. Answer the following for the polynomial: \( f(x) = x^4 - 2x^2 + 1 \). (12 pts)

(a) Use arrow notation or indicate on a graph the end behavior of \( f(x) \).

Solution:

\( f(x) \Rightarrow \infty \text{ as } x \Rightarrow \infty \) and \( f(x) \Rightarrow \infty \text{ as } x \Rightarrow -\infty \)

(b) Find all \( x \) and \( y \) intercepts.

Solution:

\( x \) intercepts: when \( y = 0 \), \( x^4 - 2x^2 + 1 = 0 \) and \( (x^2 - 1)^2 = 0 \) which gives \( (x - 1)^2(x + 1)^2 = 0 \). The intercepts are \((-1,0) \) and \((1,0) \).

\( y \) intercept: when \( x = 0 \), \( y = 1 \). The intercept is \((0,1) \).

(c) For each \( x \) intercept you found in part (b), does the graph cross or touch the \( x \)-axis at the intercept?

Solution:

The \( x \)-coordinate of each intercept is a zero of the polynomial. From part (b) we see the multiplicity of the zeros \( x = -1, 1 \) is two for each, so the graph will touch and not cross at each of these zeros.

4. Consider the polynomial \( P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4 \). (10 pts)

(a) Use the rational roots theorem to list all possible rational zeros of the polynomial.

Solution:

\( x = \pm 1, \pm 2, \pm 4 \)

(b) Now find all zeros of the polynomial over the complex numbers. In other words, zeros may be real or imaginary numbers.

Solution:

By using synthetic division we can see that \( x = 1 \) is a zero of the polynomial. From the results of synthetic division we can write:

\[ P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4 = (x - 1)(x^3 - x^2 + 4x - 4) \] and factor by grouping gives us:

\[ (x - 1)(x^3 - x^2 + 4x - 4) = (x - 1)(x^2(x - 1) + 4(x - 1)) = (x - 1)(x - 1)(x^2 + 4) = (x - 1)^2(x + 2i)(x - 2i) \]. So the zeros are \( x = 1, -2i, 2i \).
5. The graph of a parabola is shown below that crosses through $(0, -1)$ and has vertex $(2,1)$. Write down the equation of the parabola in form $y = a(x - h)^2 + k$. (5 pts)

![Parabola Graph](image)

Solution:

The vertex is given by $(2,1)$ so $h = 2$ and $k = 1$, so we get $y = a(x - 2)^2 + 1$. Since the curve crosses through $(0, -1)$ then we can find $a$ by: $-1 = a(0 - 2)^2 + 1$ and $-1 = 4a + 1$. Solving for $a$ we get $a = -\frac{1}{2}$.

The final equation is given by $y = -\frac{1}{2} (x - 2)^2 + 1$

6. For each part you are given either the graph of a function or the equation of a function. Determine whether the given graph or function is odd, even, or neither. (9 pts)

(a) You do not need to show work for this part.

![Function Graph](image)

Solution:

Neither
(b) You do not need to show work for this part.

\[ f(x) = 8x^4 - 3x^2. \] Show work to justify your Answer for this part.

Solution:

\[ f(-x) = 8(-x)^4 - 3(-x)^2 = 8x^4 - 3x^2 = f(x). \] So \( f(x) \) is even.

7. Find the slant asymptote for \( R(x) = \frac{x^3 - 4x^2 + 2x - 2}{x^2 + 2x - 1} \) (5 pts).

Solution:

After utilizing long division, the slant asymptote is found to be \( y = x - 6 \).

8. A piece of wire 14 inches long is bent into the shape of a rectangle having width \( x \) and length \( y \). Setup but do not solve an equation to find the maximum area of the rectangle as a function of either the width or the length. (5 pts)

Solution:

The wire can be sub-divided into four lengths, two of length \( x \) and two of length \( y \). The area of the resulting rectangle is given by \( A = xy \) and the length of the wire can be expressed as \( 14 = 2x + 2y \). Solving the length equation for \( x \) we get \( x = 7 - y \) and substituting into the area equation we get the answer \( A = y(7 - y) \).
9. Consider the following rational function: \( r(x) = \frac{2x^2 + 4x - 16}{x^2 - x - 2} \) (18 pts)

(a) Find all holes and/or intercepts of \( r(x) \).

Solution:

\[
    r(x) = \frac{2x^2 + 4x - 16}{x^2 - x - 2} = \frac{2(x - 2)(x + 4)}{(x - 2)(x + 1)} = \frac{2(x + 4)}{(x + 1)} \quad \text{when} \quad x \neq 2.
\]

So there is a hole at \( x = 2 \) with \( y \)-coordinate given by \( \frac{2(2 + 4)}{(2 + 1)} = \frac{12}{3} = 4 \) and the hole is located at \((2, 4)\).

The intercepts are: \((-4, 0)\) and \((0, 8)\)

(b) Find all vertical asymptotes of \( r(x) \). If there are none write NONE.

There is a vertical asymptote at \( x = -1 \).

(c) Find any horizontal and/or slant asymptotes of \( r(x) \). If there are none write NONE.

\[
    \frac{2(x + 4)}{(x + 1)} \Rightarrow \frac{2x}{x} = 2 \quad \text{as} \quad x \Rightarrow \pm \infty \quad \text{so there is a horizontal asymptote} \quad y = 2.
\]

(d) Sketch the graph of \( r(x) \). Plot any extra points as needed to confirm the graph. Label all intercepts, holes, and asymptotes as relevant.