INSTRUCTIONS: Books, notes, and electronic devices are not permitted. The exam is worth 100 points. Simplify and box all your answers. Write neatly and show all work. A correct answer with incorrect or no supporting work may receive no credit.

Name:______________________________  Instructor:__________________________

1. (28 pts)
   (a) Multiply and simplify completely: \((x^2 + 1)^2\)

   Solution:
   \[
   (x^2 + 1)^2 = (x^2 + 1)(x^2 + 1) = x^4 + x^2 + x^2 + 1 = x^4 + 2x^2 + 1
   \]

   (b) Simplify to write as a polynomial: \((6u - 3)(u + 1) - 2u(u + 3)\)

   Solution:
   \[
   (6u - 3)(u + 1) - 2u(u + 3) = 6u^2 + 6u - 3u - 3 - 2u^2 - 6u = 4u^2 - 3u - 3
   \]

   (c) Simplify completely: \(\sqrt[3]{27x^4y^3}\)

   Solution:
   \[
   \sqrt[3]{27x^4y^3} = 3xy\sqrt[3]{x}
   \]

   (d) Factor completely: \(2x^2 - 9x + 10\)

   Solution:
   \[
   2x^2 - 9x + 10 = 2x^2 - 4x - 5x + 10 = 2x(x - 2) - 5(x - 2) = (x - 2)(2x - 5)
   \]

   (e) Simplify: \(\frac{x^{-3}y^2z^{1/2}}{z^{-1/2}}\)

   Solution:
   \[
   \frac{x^{-3}y^2z^{1/2}}{z^{-1/2}} = \frac{y^2z^{1/2}z^{1/2}}{x^3} = \frac{y^2z^{(1/2+1/2)}}{x^3} = \frac{y^2z}{x^3}
   \]
(f) Combine into a single fraction: \[ \frac{1}{2x} + \frac{2}{3x^2} \]

**Solution:**

\[
\frac{1}{2x} + \frac{2}{3x^2} = \frac{3x}{6x^2} + \frac{4}{6x^2} = \frac{3x + 4}{6x^2}
\]

(g) Simplify the expression: \[ \frac{1 + \frac{2}{x^2}}{\frac{4}{x} + 3} \]

**Solution:**

\[
\frac{1 + \frac{2}{x^2}}{\frac{4}{x} + 3} = \frac{\frac{x^2 + 2}{x^2} + \frac{2}{x}}{\frac{4 + 3x}{x}} = \frac{x^2 + 2}{4 + 3x} \cdot \frac{x}{x^2} = \frac{x^2 + 2}{x(4 + 3x)}
\]

2. Solve the following equations: (25 pts)

(a) \[ x^{3/2} = 64 \]

**Solution:**

\[
x^{3/2} = 64 \quad (1)
\]
\[
x = 64^{2/3} \quad (2)
\]
\[
x = 4^2 \quad (3)
\]
\[
x = 16 \quad (4)
\]

(b) \[ -3 + x + \sqrt{x + 3} = 0 \]

**Solution:**

\[
-3 + x + \sqrt{x + 3} = 0 \quad (5)
\]
\[
\sqrt{x + 3} = 3 - x \quad (6)
\]
\[
x + 3 = (3 - x)^2 \quad (7)
\]
\[
x + 3 = 9 - 6x + x^2 \quad (8)
\]
\[
x^2 - 7x + 6 = 0 \quad (9)
\]
\[
(x - 6)(x - 1) = 0 \quad (10)
\]

So \( x = 1, 6 \) are possible solutions. Checking these solutions in the original equation we see that \( x = 1 \) is the only solution.
(c) \(x^2 - 4x - 2 = 0\)

**Solution:**

\[
x^2 - 4x - 2 = 0 \tag{11}
\]

\[
x = \frac{-(\pm 4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)} \tag{12}
\]

\[
x = \frac{4 \pm \sqrt{16 + 8}}{2} \tag{13}
\]

\[
x = \frac{4 \pm \sqrt{24}}{2} \tag{14}
\]

\[
x = \frac{2(2 \pm \sqrt{6})}{2} \tag{15}
\]

\[
x = 2 \pm \sqrt{6} \tag{16}
\]

(d) \(|x - 1| = 2\)

**Solution:**

\[
|x - 1| = 2 \tag{17}
\]

\[
\Rightarrow x - 1 = 2 \text{ and } x - 1 = -2 \tag{18}
\]

\[
\Rightarrow x = 3 \text{ and } x = -1 \tag{19}
\]

(e) Solve for \(r\): \(4r = \frac{t^2}{r}\)

**Solution:**

\[
4r = \frac{t^2}{r} \tag{20}
\]

\[
4r^2 = t^2 \tag{21}
\]

\[
r^2 = \frac{t^2}{4} \tag{22}
\]

\[
r = \pm \sqrt{\frac{t^2}{4}} \tag{23}
\]

\[
r = \pm \frac{|t|}{2} \text{ or } r = \pm \frac{t}{2} \tag{24}
\]
3. (12 pts)

(a) Plot the two points: \(A(2, 3), B(-3, -2)\) on a graph. Label points and tick marks on graph.

Solution:

(b) Find the distance between the two points.

Solution:

\[
d = \sqrt{(-3 - 2)^2 + (-2 - 3)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}
\]

(c) Find the midpoint between the two points.

Solution:

\[
\left(\frac{-3 + 2}{2}, \frac{-2 + 3}{2}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right)
\]

4. Find the domain of the following functions. Give all answers in interval notation. (15 pts)

(a) \(f(x) = \sqrt{x^2 - 3x - 10}\)

Solution:

\[
x^2 - 3x - 10 \geq 0 \\
(x - 5)(x + 2) \geq 0
\]

Note that \((x - 5)(x + 2) = 0\) when \(x = -2, 5\). By testing points we get the domain of:
\((-\infty, -2] \cup [5, \infty)\).
(b) \( h(x) = \sqrt[3]{1 - x} \)

**Solution:**

Since for any \( x \)-value, \( \sqrt[3]{1 - x} \) computes a real number, then the domain is: \((−∞, ∞)\).

(c) \( g(x) = \frac{x - 4}{x^2 - 16} \)

**Solution:**

\( x^2 - 16 = 0 \) when \( x = ±4 \) so the domain is: \((−∞, −4) \cup (−4, 4) \cup (4, ∞)\).

5. Consider the graph of the function \( f \) and answer the questions below. When appropriate give answers in interval notation. (10 pts)

(a) What is the domain of \( f \)?

**Solution:**

The domain is: \([-3, 4]\).

(b) What is the range of \( f \)?

**Solution:**

The range is: \([-2, 2]\).

(c) Find \( f(1) \)

**Solution:**

\( f(1) = 0 \).

(d) All \( x \) such that \( f(x) = 1 \).

**Solution:**

\( f(x) = 1 \) when \( x = −1, \frac{1}{2}, 2 \).

(e) All \( x \) such that \( f(x) \leq 1 \).

**Solution:**

The range is: \([-3, −1] \cup \left[ \frac{1}{2}, 2 \right] \).
6. Find an equation of a circle that satisfies center \( C \left( \frac{-1}{2}, 2 \right) \) and radius \( r = 3\sqrt{2} \). (5 pts)

Solution:

\[
\left( x - \left( \frac{-1}{2} \right) \right)^2 + (y - 2)^2 = (3\sqrt{2})^2 \text{ and the simplified form is } \left( x + \frac{1}{2} \right)^2 + (y - 2)^2 = 18
\]

7. Sketch the graph of: \( y = \sqrt{9 - x^2} \). Make sure to label tick marks on the axes. (5 pts)

Solution:

![Graph of \( y = \sqrt{9 - x^2} \)]

Potentially useful equations:

(i) \( a^3 - b^3 = (a - b)(a^2 + ab + b^2) \)

(ii) \( a^3 + b^3 = (a + b)(a^2 - ab + b^2) \)

(iii) Difference quotient of \( f(x) \): \( \frac{f(x + h) - f(x)}{h} \)