1. (20 points) Consider the quadratic function $f(x) = -2x^2 + 16x - 26$

   (a) Use the quadratic formula to find the real zeros (aka x-intercepts) of $f(x)$.

   
   \[
   a = -2 \\
   b = 16 \\
   c = -26
   \]
   
   \[
   x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-16 \pm \sqrt{16^2 - 4(-2)(-26)}}{2(-2)}
   \]
   
   \[
   = \frac{-16 \pm \sqrt{256 - 208}}{-4} = \frac{-16 \pm \sqrt{48}}{-4} = \frac{-16 \pm 4\sqrt{3}}{-4} = 4 \pm 4\sqrt{3}
   \]

   (b) Does this function have a MAXIMUM or MINIMUM value? Circle one.

   \[4 \pm 4\sqrt{3}\]

   (c) Find the vertex of the parabola that this function represents.

   \[
   x = \frac{-b}{2a} = \frac{-16}{2(-2)} = \frac{16}{-4} = 4
   \]

   \[
   f(4) = -2(4)^2 + 16(4) - 26
   \]

   \[
   = -32 + 64 - 26
   \]

   \[
   = 32 - 26
   \]

   \[
   = 6
   \]
2. (20 points) Given the function below, please answer the following questions:

(a) As \( x \to 2^- \), \( f(x) \to \) \( \boxed{3} \)

(b) As \( x \to 2^+ \), \( f(x) \to \) \( \boxed{2} \)

(c) Write the function definition for this piecewise function:

\[
f(x) = \begin{cases} 
\frac{1}{x} & x \leq 1 \\
3 & 1 < x \leq 2 \\
x & x > 2
\end{cases}
\]

(d) The point \( P(1, 1) \) is on the graph of \( f(x) \) as shown above. Find the corresponding point on the graph of \( y = f(x - 4) + 2 \)

- Shift Right by 4
- Shift Up by 2

\((1 + 4, 1 + 2) = (5, 3)\)
3. (20 points) Consider the rational function: 
\[ r(x) = \frac{2x^2 + x - 6}{x^2 + 3x + 2} \]

(a) Find the coordinates of any holes in the graph of \( r(x) \). If there are none, state NONE.

\[ \text{hole(s): } (-2, 7) \]

\[ r(x) = \frac{(2x - 3)(x + 2)}{(x + 2)(x + 1)} = \frac{2x - 3}{x + 1} \]
\[ r(-2) = \frac{2(-2) - 3}{-2 + 1} = \frac{-4 - 3}{-1} = \frac{-7}{-1} = 7 \]

(b) Identify any vertical asymptotes in the graph of \( r(x) \). If there are none, state NONE.

\[ \text{vertical asymptote(s): } x = -1 \]

(c) Identify any horizontal asymptotes in the graph of \( r(x) \). If there are none, state NONE.

\[ \text{horizontal asymptote(s): } y = 2 \]

(d) Sketch the graph of this function. Label any and all important features of the graph. [i.e. holes, intercepts, asymptotes, etc.]

\[ x\text{-intercept } ? \]
\[ 2x - 3 = 0 \]
\[ x = \frac{3}{2} \]

\[ y\text{-intercept } ? \]
\[ \frac{2(0) - 3}{0 + 1} = -3 \]
4. (20 points)

(a) Find ALL zeros of \( f(x) = x^4 + 7x^2 - 144 \)

\[
\begin{align*}
  f(x) &= x^4 + 7x^2 - 144 \\
  &= (x^2 + 16)(x^2 - 9) \\
  &= (x + 4i)(x - 4i)(x - 3)(x + 3)
\end{align*}
\]

Zeros: \( x = \pm 4i, \pm 3 \)

(b) Find ALL zeros of \( f(x) = x^3 - 6x^2 + 11x - 6 \)

\[
\begin{array}{c|cccc}
  & 1 & -6 & 11 & -6 \\
\hline
  1 &  & 1 & -5 & 6 \\
  & 1 & -5 & 6 & 0 \\
\end{array}
\]

\[
\begin{align*}
  f(x) &= (x-1)(x^2 - 5x + 6) \\
  &= (x-1)(x-3)(x-2)
\end{align*}
\]

Zeros: \( x = 1, 2, 3 \)

(c) Write an equation for a polynomial function that has the following characteristics: as \( x \to \infty \), \( f(x) \to -\infty \), as \( x \to -\infty \), \( f(x) \to \infty \). The polynomial is 5th degree with one real zero at \( x = 2 \)

\[
\begin{align*}
  f(x) &= -(x - 2)^5 \\
\end{align*}
\]
5. (20 points) Answer the following questions by CIRCLING either TRUE or FALSE. For this problem only, you do not need to justify your answer.

(a) **TRUE** / **FALSE** The y-intercept of \( f(x) = (x-a)^2(x-b)(x-c) \) is \(-a^2bc\).

(b) **TRUE** / **FALSE** The function \( f(x) = \frac{2(x-3)(x-1) + 1}{x-1} \) has an oblique asymptote of \( y = 2x - 6 \).

(c) **TRUE** / **FALSE** The domain of \((f \circ g)(x)\) given \( f(x) = \frac{2}{x^2} \) and \( g(x) = \frac{3}{x} \) is \( x \neq \frac{3}{2} \).

(d) **TRUE** / **FALSE** The expression \( 3x^2 - 2x - 11 \) has a zero in the interval \([2, 3]\)

\[
3(z)^2 - 2(z) - 11 = 12 - 4 - 11 = -3 < 0
\]

\[
3(\bar{z})^2 - 2(\bar{z}) - 11 = 27 - 6 - 11 = 10 > 0
\]