Radial Basis Functions and Pseudospectral Methods

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APPM 7400-002, MWF 12:00-12:50 pm, ECCR 139.

Course Description:

Radial Basis Functions (RBF) and RBF-generated finite differences (RBF-FD) are powerful methodologies especially for solving PDEs to high accuracy in non-trivial geometries, with irregularly shaped material interfaces, and/or when spatially variable resolution is required. In several recent large-scale benchmark tests in fluid mechanics, the geosciences, and for seismic wave modeling, the methods have been shown to outperform all previous alternatives.

The first book that covers both RBF and RBF-FD was published by SIAM in 2015 (B. Fornberg and N. Flyer: A Primer on Radial Basis Functions with Applications to the Geosciences) and will be used as course text. This book will be made available in .pdf format (as an alternative to purchasing a hard copy).

While this course technically is an upper level graduate course, it will not require any special prerequisites (beyond undergraduate calculus and diff. eq.; while some previous numerics experience could be helpful, this is not required). The course is designed not only for students specializing in numerics, but also for students who want to solve problems in different areas and would like an up-to-date survey-style overview of RBF methods. Our focus will be on how, when, and why they work, more by means of examples and heuristic explanations than by rigorous arguments. Course assignments will be mostly take the form of projects and presentations.

Brief Overview of the main Course Topics:

The course will begin by brief introductions to finite difference (FD) and pseudospectral (PS) methods, and then describe how these naturally have led to RBF and RBF-FD methods.

Finite difference methods: These were first proposed for solving PDEs in 1910, and they have remained a dominant methodology ever since. Generally, they are easy to implement, but they are more restrictive in terms of geometric flexibility than, for example, finite elements.

Pseudospectral methods: For applications in simple geometries (long intervals in 1-D, rectangular or circular domains in 2-D, periodic boxes in 3-D, spherical shells, etc.), it was noted in the early 1970's that the infinite order of accuracy limit of FD methods exists and that it sometimes can offer spectacular computational efficiencies. Another way to arrive at the same PS methods is via expansions in orthogonal functions. These PS methods soon became prominent for solving certain PDEs in areas including fluid dynamics (such as turbulence modeling), weather forecasting, long time evolution of linear and nonlinear waves, and computational electromagnetics.

Radial basis functions: RBF approximations can be seen as a generalization of PS methods, offering complete geometric flexibility and easily implemented local node refinement in any number of dimensions, while still preserving the spectral accuracy. In several recent large-scale 3-D geophysical flow applications, RBF-based codes in Matlab on a standard PCs have competed favorably against all previous methodologies, even when these were implemented on supercomputer systems. Other emerging application areas for RBF-FD are tsunami modeling and mathematical biology (e.g. solving PDEs on curved membranes).

Radial basis function-generated FD methods: Using RBFs to create generalized FD methods might offer the best opportunity yet for combining the strengths of all the previous approaches. RBF-FD methods are particularly well suited to parallel computing, since all approximations (like the case for classical FD methods) are spatially local. Development work on RBF-FD methods is under way at several research centers.