Radial Basis Functions with Applications to the Geosciences

Instructor:
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APPM 7400-005, "Open Topics: Applied Mathematics"
MWF 12:40- 1:30 pm, Meets Remotely (Zoom).

Course Description:

Radial Basis Functions (RBF) and RBF-generated finite differences (RBF-FD) are powerful methodologies for solving PDEs to high accuracy in non-trivial geometries, with irregularly shaped material interfaces, and/or when spatially variable resolution is required. In several recent large-scale applications, these methods have been shown to outperform all previous alternatives.

While this course is listed as an upper level graduate course, it will not require any special prerequisites beyond undergraduate calculus and differential equations; some previous numerics experience can be helpful. The course is designed for students who want to solve problems described by PDEs in different application areas, and would like to see this topic surveyed. Our focus will be on how, when, and why RBF-based approaches works, more by means of examples and heuristic explanations than by rigorous analysis. Course assignments will mostly take the form of projects and presentations.

The course text book (B. Fornberg and N. Flyer: *A Primer on Radial Basis Functions with Applications to the Geosciences*, SIAM 2015) will be made available electronically.

Brief Overview of the main Numerical Topics:

The numerical analysis side of course will describe the following evolution of methods:

Finite difference methods: First applied to PDEs in 1910, this remains a dominant methodology. FD approximations are easy to implement, but suffer from geometric inflexibility.

Pseudospectral methods: It was noted in the early 1970's that the infinite order of accuracy limit of FD methods exists and that it, in simple geometries, sometimes can offer spectacular computational efficiencies. PS methods have been highly successful in numerous areas, including fluid dynamics (such as turbulence modeling), weather forecasting, long time evolution of linear and nonlinear waves, and computational electromagnetics.

Radial basis functions: RBF approximations can be seen as a far reaching generalization of PS methods, offering complete geometric flexibility and easily implemented local node refinement in any number of dimensions, while still being accurate to high orders. In several recent large-scale 3-D geophysical flow applications, RBF-based codes in Matlab on a standard PCs have competed favorably against all previous methodologies, even when these were implemented on supercomputer systems. Additional emerging application areas include tsunami modeling and mathematical biology.

Radial basis function-generated FD methods: Using RBFs to create generalized FD methods offer the best opportunity yet for combining the strengths of all the previous approaches. RBF-FD methods are particularly well suited to parallel computing, since all approximations (like the case for classical FD methods) are spatially local.