APPM 2350 - Calculus 3

Course Objectives: This course extends the ideas of single-variable calculus to functions of several variables.

The main topics covered include:

- Multivariable Calculus
- Vector Analysis
- The Fundamental Theorem of Line Integrals and theorems of Gauss, Green and Stokes.

This class will form the basis of your set of everyday working skill required for math, engineering and the sciences.

Textbook: *Essential Calculus*, 2nd Edition by James Stewart. We will cover Chapters 10-13. You will also need an access code for WebAssign's online homework system. The access code can also be purchased separately.

Schedule and Topics Covered

Day	Section/Pages	Topics
1	10.1	Intro to 3D Coordinates and Graphing
2	10.2	Vectors
3	10.3	Dot Product, Projections and Work
4	10.4	Cross Product and Torque
5	10.5a/10.7	Lines in Space
6	10.7/10.8a	Vector Functions/TNB Frame
7	10.8b	Arclength & Curvature
8	10.8b/10.9	Curvature cont'd/ Motion in Space: Velocity and Decomposition of Acceleration
9	10.9 cont'd	Motion in Space
10	10.5b	Planes and cylinders
11	10.6	Quadric Surfaces
12	11.1	Multivariable Functions
13	REVIEW	Exam 1 Review
13.5	11.2* (Recitation)	Limits and Continuity
14	11.3	Partial Derivatives
15	11.4	Linear Approximations, Tangent Planes and Differentials
16	Handout	Taylor Series and Taylor's Formula for 2 Variables
17	Handout/11.5	Taylor Series Cont'd; Multivariable Chain Rule
18	11.5/11.6	Multivariable Chain Rule; Directional Derivative/Gradient
19	11.6 cont'd	Directional Derivative/Gradient
20	11.7	Classifying extrema
21	11.7/11.8	Max/Min on Closed Bounded Regions; Lagrange Multipliers
22	11.8 cont'd	Lagrange Multipliers cont'd;
23	12.1	Double Integrals over Rectangular Regions
24	12.2	Double Integrals over General Regions; Double Integrals in Polar Coordinates
25	REVIEW	Exam 2 Review
25.5	12.4* (Recitation)	Mass/Moments/Average Value
26	12.3	Double Integrals in Polar Coordinates
27	12.8	Change of Variables for Double Integrals
28	12.5	Triple Integrals
29	12.6	Cylindrical Coordinates
30	12.7	Spherical Coordinates
31	13.A (Handout)	Scalar Surface Integrals
32	13.A /13.B (pp 767-774)	Scalar Surface Integrals cont'd.; Scalar Line Integrals (pp 767-774)
33	13.B cont'd	Scalar Line Integrals cont'd. (pp 767-774)

34	13.C (pp 761-765, 774-776)	Vector Fields (pp 761-765) and Line Integrals of Vector Fields: Flow (pp 774-776)
35	13.C/13.D1 (Handout)	Line Integrals of Vector Fields: Flow cont'd; /Flux through a curve
36	13.D2 (Handout)	Flux through a surface
37	REVIEW	Exam 3 Review
38	13.E1 (pp 798-799, 800-801)	Divergence (pp 798-799) and Green's Theorem (Flux/Divergence form) (pp 800-801)
39	13.E2 (pp 829-834)	Gauss' Theorem (pp 829-834)
40	13.F1 (pp 795-798, 788-793)	Curl (pp 795-798) and Green's Theorem (Circulation/Curl form) (pp 788-793)
41	13.F2 (pp 824-828)	Stokes' Theorem (pp 824-828)
42	13.G (779-785)	Fundamental Theorem for Line Integrals (779-785)
43	REVIEW	Final Exam Review

Prerequisites:

- ONE of the following courses (minimum grade C-): APPM 1360 or MATH 2300

Equivalent Courses: Duplicate Degree Credit Not Granted:

- MATH 2400

Learning Objectives by Section

Section	Topics	Learning Objectives – After completing this section, students should be able to do the following:
10.1	3-D Coordinates	• Represent points in 3-dimensions. • Know the right-hand rule for relating coordinate axes in 3-dimensions. • Understand the notation $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3, \ldots \mathbb{R}^n$. • Derive and use the distance formula in three dimensions, and be able to generalize to n dimensions. • Give the equation for a sphere or ball and recognize the generalization from two dimensions. • Use traces to sketch basic surfaces in 3-dimensions (cylinders, paraboloids, cones, sphere, ellipsoids) given their equation.
10.2	Vectors	• State the definition of a vector • Calculate the magnitude of a vector • Find unit vectors • Multiply vectors by scalars algebraically and geometrically. • Add and subtract vectors algebraically and geometrically. • Write vectors using the standard basis vectors. • Use vectors to find the resultant force on an object.

10.3	Dot Product	• State the algebraic definition and explain the geometric meaning of the dot product. • Compute dot products using both forms of the dot product. • Use dot products to compute the angle between vectors. • Find orthogonal projections. • Find scalar projections. • Find orthogonal decompositions. • Use the dot product to calculate work.
10.4	Cross Product	• Define the cross product • Compute cross products and understand the resulting space a cross product lives in • State key properties of cross products • Relate area to magnitudes of cross products • Define the scalar triple product and explain what it means geometrically • Use cross products to calculate torque
10.5a	Lines in Space	• Find the equation for a line in space using 3 different representations: parametric equations, symmetric equations and vector-valued functions • Determine if 2 given lines are parallel, skew or intersecting • Find the shortest distance from a point to a line using cross products.
10.5b	Planes in Space	• Find the equation of a plane given either 3 points on the plane or a point on the plane and a vector normal to the plane. • Given the equation of a plane, find a vector normal to the plane. • Find the shortest distance between a plane and a point. • Find the distance between two parallel planes • Find the angle between two planes • Find the line of intersection of 2 planes. • Find the point of intersection of a line and a plane.

10.6	Cylinders and Quadric Surfaces	• Generalize the definition of a cylinder • Be able to sketch various cylinders in 3D • Define and sketch various quadric surfaces using traces • Identify and sketch the different traces for different quadric surfaces. • Recognize when level set and graph-of-function formula are quadric surfaces.
10.7	Vector Valued Functions and Space Curves	• Define vector-valued functions and interpret geometrically as a position function. • Relate parametric curves to the space curve defined by a vector-valued function • Compute the derivative of a vector-valued function, and interpret geometrically as velocity. • Calculate integrals of vector-valued functions. • Find the unit tangent vector given a vector-valued function and interpret geometrically • Find the unit normal vector given a vector-valued function and interpret geometrically • Find the unit binormal vector given a vector-valued function and interpret geometrically.
10.8	Arclength and Curvature	• Compute the length of a parametric curve. • Define the arclength function. • Parameterize a curve in terms of arc length. • Define and compute curvatures of various curves • Interpret curvature in terms of an osculating circle • *[Coverage of torsion is optional here].*
10.9	Motion in Space: Velocity and Acceleration	• Decompose the acceleration vector into tangential and normal components. • Define and use the cross-product formula for curvature • Use vector-valued functions to analyze projectile motion and find forces on moving objects. • *[Skip section on Kepler's Laws of Planetary Motion].*

11.1	Multivariable Functions	• Define a function of several variables. • Find the domain and range of a function of several variables. • Represent a function of several variables graphically and by using contour plots. • Sketch level curves for functions of 2 variables and level surfaces for functions of 3 variables.
11.2	Limits and Continuity	• Informally explain the concept of the limit of a function of two variables. • Apply theorems that guarantee limits exist. • Understand the two path criterion to show that a limit does not exist and apply it to solve problems about limits. • Determine continuity of functions of several variables.
11.3	Partial Derivatives	• Calculate first and second partial derivatives. • Recognize various notation for partial derivatives. • Provide geometrical meaning of partial derivatives. • Provide geometrical meaning of second partial derivatives, • Estimate partial derivatives from tables. • Estimate partial derivatives from a set of level curves. • Compute higher order partial derivatives. • Understand graphical interpretation of 2nd order partial derivatives. • Verify whether a function satisfies a given partial differential equation.
11.4	Tangent Planes, Linear Approximations and Differentials	• Find explicit formulas for tangent planes. • Understand the relationship between linear approximation and the tangent plane. • Understand the definition of differentiability for multivariable functions. • Find linear approximations of functions of 3 or more variables • Use linearization to estimate values of the function near a point of tangency. • Calculate differentials of functions of 2 or 3 variables. • Use differentials to approximate changes in functions of 2 or 3 variables.

11.5	The Multivariable Chain Rule	• Define the chain rule for functions of more than one variable • Compute the derivative of the composition of a function of several variables with a vector-valued function. • Use the chain rule of several variables to implicitly differentiate a curve.
11.6	Directional Derivatives and the Gradient Vector	• Compute directional derivatives. • Interpret directional derivatives graphically • Estimate directional derivatives given level curves • Use the directional derivative to show that the gradient vector points in the direction of greatest increase for the function. • Find the largest and smallest derivatives of a function at a given point. • Recognize that the gradient points in the initial direction of greatest increase of the function's value. • Recognize that the gradient is normal to level sets.
11.7	Maximums and Minimums	• Find the critical points of a function of two variables. • Use the 2nd derivative test to classify critical points of a function of two variables. • Define a constrained optimization problem. • Optimize a function on a closed and bounded set by parameterizing the boundary.
11.8	Lagrange Multipliers	• Understand the geometric basis of the method of Lagrange multipliers • Use Lagrange multipliers to solve constrained optimization problems. • *[Skip Lagrange multipliers with two constraints].*
12.1	Double Integrals over Rectangular Regions	• Understand the Riemann sum definition of a double integral, both algebraically and geometrically. • Use iterated integrals to compute double integrals. • Apply Fubini's Theorem. • Use double integrals to calculate volumes.

12.2	Double Integrals over General Regions	• Generalize the discussion of double integrals to now be defined over complex domains • Compute definite integrals over generic domains • Apply Fubini's theorem to general domains of integration • Use double integrals to calculate areas and/or volumes. • Sketch the region of integration given a double integral. • Change the order of integration given a double integral.
12.3	Double Integrals in Polar Coordinates	• Compute double integrals in polar coordinates. • Identify when and how to change integrals to polar coordinates • Convert polar double integrals back to Cartesian double integrals. • Understand algebraically and geometrically how $$dA$$ is calculated in polar coordinates.
12.4	Applications of Double Integrals	• Use double integrals to calculate average value. • Use double integrals to calculate mass and center of mass of 2D objects. • Use double integrals to calculate volume • Use double integrals to calculate area.
12.5	Triple Integrals	• Understand the Riemann sum definition of a triple integral, both algebraically and geometrically. • Use iterated integrals to compute triple integrals. • Use triple integrals to calculate volumes. • Use triple integrals to calculate average values. • Use tripled integrals to calculate mass and center of mass of 3D objects.

12.6	Triple integrals in Cylindrical Coordinates	• Convert triple integrals from Cartesian to Cylindrical coordinates (and vice versa). • Identify when cylindrical coordinates will simplify a triple integral. • Calculate volume, mass, center of mass and average values by setting up integrals in cylindrical coordinates when appropriate. • Understand both algebraically and geometrically how dV is calculated in cylindrical coordinates.
12.7	Triple integrals in Spherical Coordinates	• Convert triple integrals from Cartesian to Spherical coordinates (and vice versa). • Convert triple integrals from Cylindrical to Spherical coordinates (and vice versa). • Identify when spherical coordinates will simplify a triple integral. • Calculate volume, mass, center of mass and average values by setting up integrals in spherical coordinates when appropriate. • Understand both algebraically and geometrically how dV is calculated in spherical coordinates.
12.8	Change of Variables in Multiple Integrals	• Understand how to extend the substitution rule from Calculus 1 to change variables in double integrals. • Define the Jacobian to compute the change of variables for integration for both double integrals and triple integrals. • Practice identifying good transformations to use to make double integrals easier • *[Skip general coordinate changes for triple integrals].*
13.A	Scalar Surface Integrals	• Compute surface area using double integrals. • Understand algebraically and geometrically how to convert dA when evaluating surface integrals. • Find the mass, moments and center of mass of a surface.
13.B	Scalar Line Integrals	• Calculate a scalar line integral along a curve. • Use a line integral to evaluate arclength, mass, moments and center of mass of a wire. • Understand algebraically and geometrically how to convert ds when evaluating line integrals.

13.C	Vector Line Integrals along a curve (Flow)	• Explain the concept of a vector field and make sketches of simple vector fields in the plane. • Identify radial vector fields. • Provide geometric and physical explanations of the integral of a vector field over a curve (i.e. flow, circulation). • Compute the line integral of a vector field along a curve directly. • Estimate line integrals of a vector field along a curve from a graph of the curve and the vector field. • Use a line integral to calculate work done in moving an object along a curve in a vector field. •
13.D	Flux: Across curves (line integrals) and through surfaces (surface integral)	• Compute the flux of a vector field across a curve. • Develop understanding of flux as the total flow rate across a boundary. • Relate flux across a curve boundary to flux across a surface boundary. • Compute the flux of a vector field across a surface.
13.E	Divergence, Green's Theorem (Flux-Divergence Form) and Gauss' Theorem	• State the definition of the divergence of a vector field in any dimension. • Understand how divergence measures local expansion. • State and use Green's Theorem (Flux-Divergence form) • State and use the Divergence/Gauss' Theorem (when applicable) • Interpret Gauss's Theorem as a higher dimension version of Green's Theorem (Flux-Divergence form) • Identify situations where Green's and/or Gauss' Theorem does not apply.
13.F	Curl, Green's Theorem (Curl-Circulation Form) and Stokes' Theorem	• State the definition of the curl of a vector field in two and three dimensions. • Understand how curl measures local rotation in two dimensions. • State Green's Theorem (Curl-Circulation form), and identify when to use it • Use Green's Theorem to calculate flow/work/circulation when applicable. • Interpret Stokes' theorem as a higher dimensional version of Green's theorem • Be able to use Stokes' theorem to ease computation of surface integrals or double integrals

13.G	Fundamental Theorem for Line Integrals	• Identify gradient vector fields. • Find potential functions for gradient fields • Explain the concept of a conservative field, state and apply theorems that give necessary and sufficient conditions for when a vector field is conservative, and describe applications to physics. • State the Fundamental Theorem of Calculus for line integrals and use it to simplify computations