APPM 1350 - Calculus 1

Course Objectives: This class will form the basis for many of the standard skills required in all of Engineering, the Sciences, and Mathematics.

Specifically, students will:

- Understand the concepts, techniques and applications of differential and integral calculus including limits, rate of change of functions, and derivatives and integrals of algebraic and transcendental functions.
- Improve problem solving and critical thinking

Textbook: *Essential Calculus*, 2nd Edition by James Stewart. We will cover Chapters 1-5. You will also need an access code for WebAssign's online homework system. The access code can also be purchased separately.

Schedule and Topics Covered

Day	Section	Topics
1	Appendix A	Trigonometry
2	1.1	Functions and Their Representation
3	1.2	Essential Functions
4	1.3	The Limit of a Function
5	1.4	Calculating Limits
6	1.5	Continuity
7	1.5/1.6	Continuity/Limits Involving Infinity
8	1.6	Limits Involving Infinity
9	2.1	Derivatives and Rates of Change
10	Exam 1 Review	Exam 1 Topics
11	2.2	The Derivative as a Function
12	2.3	Basic Differentiation Formulas
13	2.4	Product and Quotient Rules
14	2.5	Chain Rule
15	2.6	Implicit Differentiation
16	2.7	Related Rates
17	2.8	Linear Approximation and Differentials
18	3.1	Max and Min Values
19	3.2	The Mean Value Theorem
20	3.3	Derivatives and Shapes of Graphs
21	3.4	Curve Sketching
22	Exam 2 Review	Exam 2 Topics
23	3.4/3.5	Curve Sketching/Optimization Problems
24	3.5	Optimization Problems
25	3.6/3.7	Newton's Method/Antiderivatives
26	3.7/Appendix B	Sigma Notation
27	Appendix B, 4.1	Sigma Notation/Areas and Distance
28	4.1/4.2	Areas and Distance/The Definite Integral
29	4.3	Evaluating Definite Integrals
30	4.4	The Fundamental Theorem of Calculus
31	4.4/4.5	The Fundamental Theorem/Substitution Rule
32	4.5	Substitution Rule
33	5.1	Inverse Functions
34	Exam 3 Review	Exam 3 Topics
35	5.2	The Natural Logarithm Function
36	5.3	The Natural Exponential Function
37	5.4	General Log and Exponential Functions

38	5.5	Exponential Growth and Decay
39	5.6	Inverse Trig Functions
40	5.6/5.7	Inverse Trig/Hyperbolic Functions
41	5.7/5.8	Hyperbolic Functions/Indeterminate Forms
42	5.8	l'Hospital's Rule
43	Final Review	Cumulative

Prerequisites:

- Placement into calculus based on your admissions data and/or CU Boulder coursework
- OR any ONE of the following courses (minimum grade C-): APPM 1235, MATH 1021, MATH 1150, MATH 1160, or MATH 1300

Equivalent Courses: Duplicate Degree Credit Not Granted:

- APPM 1345, ECON 1088, MATH 1081, MATH 1300, MATH 1310, MATH 1330

Learning Objectives by Section

Section	Topics	Learning Objectives – After completing this section, students should be able to do the following:
App. A	Trigonometry	• Define trigonometric functions; understand right triangle trigonometry and the unit circle • Know and apply identities involving trigonometric functions • Plot graphs of trigonometric functions • Find the domain and range of trigonometric functions • Solve equations involving trigonometric functions
1.1	Functions and Their Representation	• State the definition of a function • Find the domain and range of a function • Perform basic operations and compositions of functions • Determine whether a function is even or odd • Work with piecewise defined functions

1.2	Essential Functions	Plot, find the domain and range, and state key properties of: • Linear Functions • Polynomials • Power Functions • Rational Functions • Trigonometric Functions • Piecewise defined functions
1.3	The Limit of a Function	• Consider values of a function at inputs approaching a given point. • Understand the concept of a limit. • Understand the precise mathematical definition of a limit using δ and ϵ. • Calculate limits from a graph (or state that the limit does not exist). • Define a one-sided limit. • Explain the relationship between one-sided and two-sided limits • Distinguish between limit values and function values.
1.4	Calculating Limits	• Calculate limits using limit laws. • Calculate limits by replacing a function with a continuous function that has the same limit. • Understand the Squeeze Theorem and how it can be used to find limit limit values. • Calculate limits using the Squeeze Theorem.
1.5	Continuity	• Identify where a function is, and is not, continuous. • Understand the connection between continuity of a function and the value of a limit. • Make a piecewise function continuous. • State the Intermediate Value Theorem including hypotheses. • Determine if the Intermediate Value Theorem applies. • Sketch pictures indicating why the Intermediate Value Theorem is true, and why all hypotheses are necessary. • Explain why certain points exist using the Intermediate Value Theorem.

1.6	Limits at Infinity	• Understand what is meant by the form of a limit. • Calculate limits of the form zero over zero. • Identify determinate and indeterminate forms. • Discuss why infinity is not a number. • Recognize when a limit is indicating there is a vertical asymptote. • Evaluate the limit as x approaches a point where there is a vertical asymptote. • Understand the relationship between limits and vertical asymptotes. • Discuss what is means for a limit to equal ∞. • Define a horizontal asymptote. • Find horizontal asymptotes using limits. • Recognize that a curve can cross a horizontal asymptote. • Produce a function with given asymptotic behavior.
2.1	Derivatives and Rates of Change	• Recognize and distinguish between secant and tangent lines. • Compute a difference quotient and take a limit of a difference quotient. • Use limits to find the slope of the tangent line at a point. • Understand the definition of the derivative at a point. • Estimate the slope of the tangent line graphically. • Compare average and instantaneous velocity.
2.2	Derivative as a Function	• Understand the derivative as a function related to the original definition of a function. • Find the derivative function using the limit definition. • Relate the derivative function to the derivative at a point. • Explain the relationship between differentiability and continuity. • Relate the graph of the function to the graph of its derivative. • Determine whether a piecewise function is differentiable.

2.3	Basic Differentiation Formulas	• Use the definition of the derivative to develop shortcut rules to find derivatives of:
		  – constants and constant multiples
		  – powers of x
		  – sums and differences of functions
		• Compute the derivative of polynomials.
		• Recognize different notation for the derivative.
		• State the derivative of the sine and cosine functions.

2.4	The Product and Quotient Rules	• Identify products of functions. • Use the product rule to calculate derivatives. • Identify quotients of functions. • Use the quotient rule to calculate derivatives. • Combine derivative rules to take derivatives of more complicated functions.
2.5	The Chain Rule	• Recognize a composition of functions. • Take derivatives of compositions of functions using the chain rule. • Understand rate of change when quantities are dependent upon each other. • Apply chain rule to relate quantities expressed with different units. • Take derivatives that require the use of multiple rules of differentiation. • Use order of operations in situations requiring multiple rules of differentiation. • Use the product, quotient and/or chain rules to calculate derivatives of trigonometric functions.
2.6	Implicit Differentiation	• Identify explicit vs implicit functions • Understand the derivatives of functions that are not defined explicitly in terms of an independent variable. • Calculate derivatives of expressions with multiple variables implicitly. • Find the equation of the tangent line for curves that are not plots of functions.

2.7	Related Rates	• Identify word problems as related rates problems. • Translate word problems into mathematical equations • Solve related rates word problems.
2.7	Linear Approximations and Differentials	• Find the linear approximation to a function at a point and use it to approximate the function value. • Identify when linear approximation can be used. • Label a graph with the appropriate quantities in linear approximation. • Compute differentials. • Contrast the notation and meaning of dy and Δy • Determine the error in using the linear approximation to a function
3.1	Max and Min Values	• Define and find critical points. • Define and find local maximum and local minimum. • Classify critical points. • State and apply the First Derivative Test. • Define and find absolute maximum and minimum. • Understand the statement of the Extreme Value Theorem.
3.2	The Mean Value Theorem	• Understand the statement of the Mean Value Theorem. • Sketch pictures to illustrate why the Mean Value Theorem is true. • Understand how Rolle's Theorem relates to the Mean Value Theorem. • Determine whether Rolle's Theorem or the Mean Value Theorem can be applied. • Find the values guaranteed by Rolle's Theorem or the Mean Value Theorem. • Compare and contrast the Intermediate Value Theorem, Mean Value Theorem and Rolle's Theorem. • Use the Mean Value Theorem to solve word problems.

3.3	Derivatives and Shapes of Graphs	• Use the first derivative to determine whether a function is increasing or decreasing. • Sketch a graph of the second derivative, given the original function. • Sketch a graph of the original function, given the graph of its first or second derivative. • Define and find inflection points • State and apply the Second Derivative Test • Sketch a graph of a function satisfying certain constraints on its higher-order derivatives. • State the relationship between concavity and the second derivative.
3.4	Curve Sketching	• Use asymptotes, intercepts, and symmetry to sketch the approximate graph • Look for intervals of increasing/decreasing value, local max/local min, concavity, and inflection points to better estimate the curve for a graph
3.5	Optimization Problems	• Recognize optimization problems. • Translate a word problem into the problem of finding the extreme values of a function. • Set-up an optimization problem by identifying the objective function and appropriate constraints. • Solve optimization problems by finding the appropriate extreme values.
3.6	Newton's Method	• Understand, derive, and implement Newton's method for finding solutions to equations • Implement Newton's method by hand • Identify when Newton's method fails to work • Explain how the equation in Newton's method works graphically.

3.7	Antiderivatives	• Define an antiderivative. • Compute basic antiderivatives. • Compare and contrast finding derivatives and finding antiderivatives. • Define initial value problems. • Solve initial value problems. • Use antiderivatives to solve word problems. • Discuss the meaning of antiderivatives of the velocity and acceleration.
Appendix B	Sigma Notation	• Express the sum of n terms using sigma notation. • Apply the properties of sums when working with sums in sigma notation. • Know and apply the formulas for the following sums:
4.1	Areas and Distances	• Understand the relationship between area under a curve and sums of areas of rectangles. • Approximate area of the region under a curve. • Compute left, right and midpoint Riemann sums. • Compute definite integrals using geometry. • Interpret the product of rate and time as an area. • Recognize Riemann sums.
4.2	The Definite Integral	• Understand how limits of Riemann sums are used to find exact area. • State the definition of the definite integral. • Evaluate definite integrals using the limit definition. • Define and approximate net area. • Split the area under a curve into several pieces to aid with calculations. • Use symmetry to calculate definite integrals.
4.3	Evaluating Definite Integrals	• Evaluate definite integrals using the Second Fundamental Theorem of Calculus • Properties of indefinite integration

4.4	The Fundamental Theorem of Calculus	• Define accumulation functions. • Calculate and evaluate accumulation functions. • State the First Fundamental Theorem of Calculus • Take derivatives of accumulation functions using the First Fundamental Theorem of Calculus • State the Second Fundamental Theorem of Calculus • Evaluate definite integrals using the Second Fundamental Theorem of Calculus. • Understand the relationship between indefinite and definite integrals. • Compute the average value of a function on an interval. • State and apply the mean value theorem for integrals.
4.5	The Substitution Rule	• Determine when a function is a composition of two or more functions. • Calculate indefinite and definite integrals requiring complicated substitutions. • Recognize common patterns in substitutions.
5.1	Inverse Functions	• Define one-to-one functions • Determine when an inverse function exists and find inverse functions. • Understand how the derivative of an inverse function relates to the original derivative graphically and algebraically • Find the derivative of an inverse function using the derivative of the original function.
5.2	The Natural Logarithmic Function	• Define the natural logarithm • Identify and utilize properties of logarithmic functions • State the derivative of the natural logarithm • Identify situations where logs can be used to help find derivatives. • Use logarithmic differentiation to simplify taking derivatives.

5.3	The Natural Exponential Function	• Define and graph the natural exponential function. • Identify and utilize properties of exponentials. • State the derivative of the natural exponential function • State the antiderivative of the natural exponential function. • Define e as a limit.
5.4	General Logarithmic and Exponential Functions	• Define and graph general logarithmic functions • Define and graph general exponential functions • Take derivatives of functions involving logs and exponents • Find antiderivatives of functions involving logs and exponents.
5.5	Exponential Growth and Decay	• Understand and solve basic differential equations • Apply differential equations to population growth and Newton's law of cooling
5.6	Inverse Trigonometric Functions	• Identify domain and range of inverse trig functions • Derive the derivatives of inverse trignometric functions • Evaluate integrals related to inverse trigonometric functions.
5.7	Hyperbolic Functions	• State the definition of hyperbolic functions • Evaluate derivatives of Hyperbolic functions • Use properties of hyperbolic functions • Hyperbolic identities • Inverse Hyperbolic functions and their derivatives
5.8	Indeterminate Forms and L'Hospital's Rule	• Determine if a form is indeterminate • Convert some indeterminate forms to the form zero over zero or infinity over infinity • Determine when L'Hopital's Rule can be used. • Use L'Hopital's rule to compute limits