Pretest
APPM 5450 Spring 2016 Applied Analysis 2

Instructor: Dr. Becker
Instructions: Take this *quickly* without spending over 30 minutes. It will not be graded and is intended to help you find weaknesses from last semester.
These are mainly True/False questions:

1. A norm is a convex function ______
2. limsup is always well-defined ______
3. If T is linear and $x_n \to x$, then $T(x_n) \to T(x)$ ______
4. If f is continuous over a compact set K, then it is also uniformly continuous ______
5. If f is continuous and G is an open set in its domain, then $f(G)$ is open as well ______
6. A separable space also compact ______
7. A compact space separable ______
8. If (f_n) is a sequence of continuous functions converging to f pointwise, then f is also continuous ______
9. If X is a metric space, then $C(X)$ is a Banach space ______
10. Polynomials are dense in $C([a, b])$ ______
11. $C([a, b])$ is separable ______
12. If K is compact, then a subset of $C(K)$ is pre-compact if it is bounded ______
13. Let X be a complete metric space and $T : X \to X$ satisfy $d(T(x), T(y)) < d(x, y)$ for all $x, y \in X$, then $\exists!\, x$ such that $x = Tx$ ______
14. Let $\mathcal{T}_1$ and $\mathcal{T}_2$ be two topologies on X, then the identity mapping $\mathcal{I} : (X, \mathcal{T}_1) \to (X, \mathcal{T}_2)$ is continuous ______
15. For which p is $\ell^p(\mathbb{N})$ a normed linear space? Banach space? Hilbert space? ______
16. $C^k([a, b])$ is a Banach space with the uniform norm? ______
17. A subspace is necessarily a closed set ______
18. A Hamel basis is such that every element of the space can be written as a finite linear combination of basis elements ______
19. Every Banach space has a Schauder basis ______
20. If X and Y are normed linear spaces and M is a dense subspace of X, and $T \in \mathcal{B}(M, Y)$, then there is a unique extension of T to all of X ______
21. Let $T \in \mathcal{B}(X, Y)$ for Banach spaces X and Y, then T^{-1} is bounded as well ______
22. The right-shift operator S on ℓ^∞ is one-to-one ______
23. The right-shift operator S on ℓ^∞ is onto ______

24. Let X and Y be Banach and $T \in \mathcal{B}(X, Y)$, then if the kernel of T is trivial, there exists $c > 0$ such that $\forall x$, $\|Tx\| \geq c\|x\|$ ______

25. Let X be a normed linear space, then its dual $X^* = \mathcal{B}(X, \mathbb{R})$ a Banach space. ______

26. $x_n \rightharpoonup x$ means $\forall \varphi \in X^*$, $\varphi(x_n) \to \varphi(x)$ ______

27. Let $(T_n) \subset \mathcal{B}(X, Y)$, then if $\|T_n - T\| \to 0$, we say (T_n) converges *strongly* ______

28. An operator $T \in \mathcal{B}(X, Y)$ is called *compact* if it maps bounded sets $B \subset X$ to compact sets $T(B) \subset Y$ ______

29. A compact operator has finite rank ______

30. In finite dimensions, weak convergence implies strong convergence ______

31. We say a sequence $(\varphi_n) \subset X^*$ converges to φ in the weak-* sense if it converges weakly with respect to X^{**}, that is, $\forall f \in X^{**}$, $f(\varphi_n) \to f(\varphi)$ ______

32. In a reflexive Banach space, the closed unit ball in X^* is weakly compact ______